Investmech: Variable amplitude loading

Dr. Michiel Heyns Pr.Eng.
T: +27 12 664-7604
C: +27 82 445-0510
mheyns@investmech.com

Objective

- Understand the analysis of dynamic loads on structures
- Scope
  - Types of loading
  - Statistical stress analysis on real structures
  - Stress collective, S-N diagram
  - Mean stress
- Expected result
  - Describe methods of counting load cycles
  - Calculate stress ratio

Types of loading

Loads can be tensile, compressive and/or shear loads (torsion) that produce stress and deformation

- Static load
  - Do not change over time
  - SANS 10160-1 – dead or permanent load
- Quasi-static load
  - Applied at rate lower than lowest natural period of the structure
    - Investmech uses a load application period of 5 x the lowest natural period (1/5th the lowest natural frequency)
- Dynamic loads
  - Load application period shorter than above
  - Deflection modes need to be taken into account
  - Shock load
    - Impact load period significantly shorter than the natural period
    - Shock waves, etc.

Causes for the loads

- Thermal
  - Normally quasi-static
  - Can be deterministic
- Process activities
  - Random, but steady-state
- Wind loads
- Cavitation
- Fluid-structural interactions
  - Water-hammer
  - Tidal – normally quasi-static of nature
  - Waves
- Mechanical loads
  - unbalance, misalignment, screen suspension, crusher supports, impact hammers, etc.
- Human interactions
  - Dropping objects, explosions, crushes, etc.
- Cluster events (Storms, process in reactors, accidents, derailing of railway vehicles, etc.)

Students to read information in the notes. Just a general introduction required.

Loads and stress-strain

- Loads are converted to stress
  - Elementary equations:
    - ❑ Normal / membrane stress: $\sigma_n = \frac{F}{A}$
    - ❑ Bending stress: $\sigma_b = \frac{M_x y}{I_{xx}} - \frac{M_y x}{I_{yy}}$. Demonstrate signs in class.
    - ❑ Shear stress (Torsion): $\tau = \frac{kV}{A} + \frac{Tr}{J}$. Note the directions!
      - Circular: $\tau_{max} = \frac{3}{2} \cdot \frac{V}{A}$; Rectangular:$\tau_{max} = \frac{4}{3} \cdot \frac{V}{A}$; Thin round tube:$\tau_{max} = 2 \cdot \frac{V}{A}$; I-beam: $\tau_{max} = \frac{V}{A_{web}}$
      - From $\tau = \frac{VQ}{Ib}$, with $Q = \int y \, dA$
  - Finite element analysis
  - Strain measurement and conversion to stress
    - ❑ Important: use the correct constitutive model:
      - E.g. linear elastic & isotropic: $\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z)$, etc.

Classification of signal types

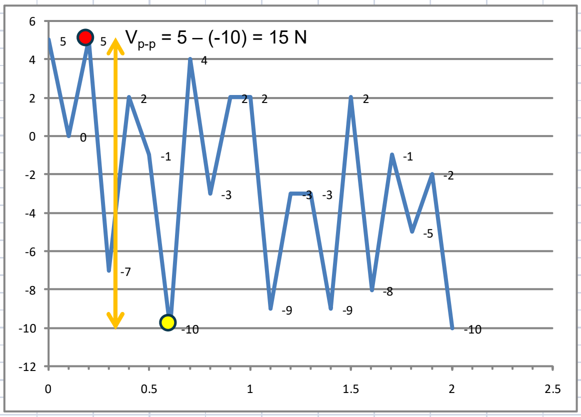
- Signal
  - Deterministic [r.m.s., peak-to-peak, mean-square analyses]
    - Periodic
      - Sinusoidal
      - Complex
    - Quasi-periodic
    - Non-periodic
      - Transient
      - Shock
  - Random
    - **Stationary** [Statistical parameters stays constant as function of time]
      - **Ergodic** [Statistical parameters and auto-correlation do not vary for time and different sample functions]
        - Self-stationary
          - Strongly
          - Weakly
      - Non-ergodic
    - Non-stationary

A signal is ergodic if its statistical properties (such as mean and variance) can be deduced from a single, sufficiently long sample of the process

For stationary signals, do time and frequency analysis. Take note of the effect of ergodic and non-ergodic effects

Measured time signal

Time [s]	F [N]	
0	5	
0.1	0	
0.2	5	
0.3	-7	
0.4	2	
0.5	-1	
0.6	-10	
0.7	4	
0.8	-3	
0.9	2	
1	2	
1.1	-9	
1.2	-3	
1.3	-3	
1.4	-9	
1.5	2	
1.6	-8	
1.7	-1	
1.8	-5	
1.9	-2	
2	-10	
Mean	**-2.33**	**N**
Std.Dev.	**5.00**	**N**



The peak-to-peak value is 5 – (-10) = 15 N in this case The mean was calculated as -2.33 N and standard deviation 5.00 N

What is the peak-value? 5 at the red dot, or -10 at the yellow dot? This depends entirely on the type of data shown. For acceleration of rotating machinery, it will be -10.

Time domain parameters

- Peak value (V_p) : maximum absolute value of the signal
  - Maximum(abs(signal(t)))
- Peak-to-peak value (V_{p_p}) : maximum – minimum value
- Mean :

$$\mu = \lim_{T \to \infty} \frac{1}{T} \int_0^T f(t)dt$$
$$= \frac{1}{N} \sum_{i=1}^{N} f(i)$$

  - For a random signal, the mean is zero

Root-mean-square

$$RMS = \sqrt{\lim_{T \to \infty} \frac{1}{T} \int_0^T (f(t))^2 \, dt}$$
$$= \sqrt{\frac{1}{N} \sum_{i=1}^{N} f(i)^2}$$

- Gives the intensity of the data which is an indication of the energy
- This is an Overall Value, that is, one value that describes the characteristic of all the values
- With band-pass filter will give narrow-band intensity

Crest factor

$$cf = \frac{Peak\ of\ the\ signal}{RMS}$$

- For a pure sine wave cf = $2^{(1/2)}$
- A cf > 3 indicates on irregularities in the signal
- The cf is not monotone
  - Will not necessarily increase with an increase in RMS
- Used to describe the “peakiness” of a function/signal

Variance and standard deviation

- Variance:

$$\sigma^2 = \lim_{T \to \infty} \frac{1}{T} \int_0^T (f(t) - \mu)^2 dt$$
$$= \frac{1}{N} \sum_{i=1}^{N} (f(i) - \mu)^2$$

- Variance = (standard deviation)^2
- This quantity gives an indication of the distribution of the data points around the mean

Kurtosis

$$KU = \frac{1}{\sigma^4} \left(\lim_{T \to \infty} \frac{1}{T} \int_0^T (f(t) - \mu)^4 dt \right)$$
$$= \frac{1}{\sigma^4 N} \sum_{i=1}^{N} (f(i) - \mu)^4$$

- The kurtosis is not monotome
- Describe the peakiness of a signal
- For sine wave KU=2
- For a random signal KU=1.5

Problem : Time domain analysis

The following sample record is given:
-1; -0,5; 0; 0,5; 1; 0,75; 0,5; 0,2; -0,2
Calculate:

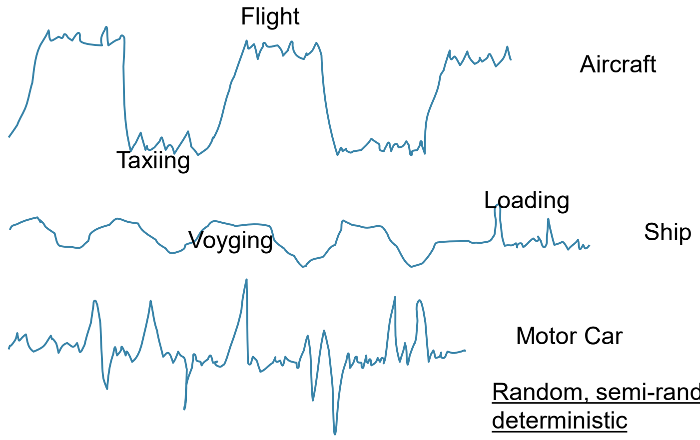
1. the peak-to-peak value,
2. the mean,
3. the rms,
4. the crest factor,

of the signal

Background

- Stress data must be accurate
- Spectrum = statistical representation of loads or stresses
- Statistical histories are used
  - ❑ Actual stress cycles cannot be known in advance
  - ❑ Stress history must be derived

Stress histories



Random, semi-random, deterministic

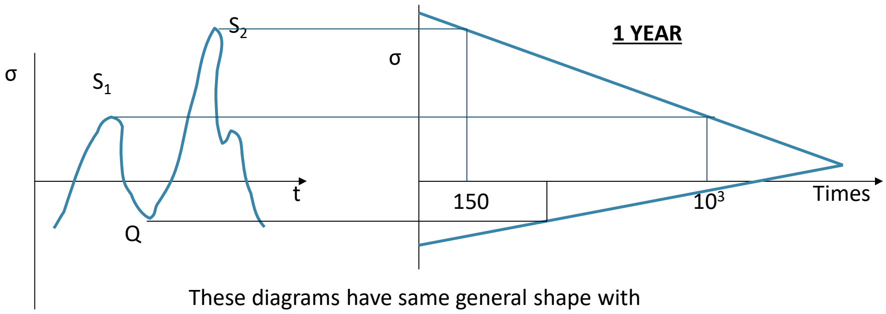
Peak-valley reduction:
To calculate high and low turning points in a signal

Obtaining load spectra

- Load spectra for stress measured or from transient analysis:
  - PSD or Spectral analysis
    - ❑ Note, the results is for a specific period and must be scaled!
  - Counting methods
    - Peak counting, Mean-crossing peak count, Range pair count, Range-pair-mean count, Rainflow count, Reservoir counting method
    - ❑ The counting method must produce the correct crack initiation and growth result
    - ❑ Counting method must detect peak, mean, minimum, and maximum of signal
    - ❑ The results are presented in a histogram for $\Delta\sigma_R$ and n_R
    - ❑ Note, the results is also only for the duration of the signal measured and must be scaled

Exceedance diagrams

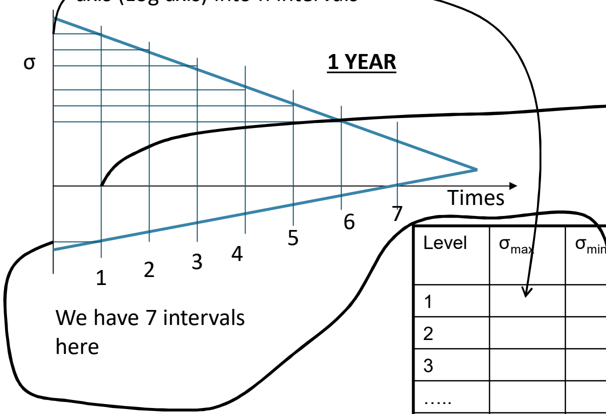
The diagram shows how many times a certain stress or load level is exceeded. The method makes use of peaks and troughs.



These diagrams have same general shape with deviations on the "straightness" of the lines.

Intervals on an exceedance diagram

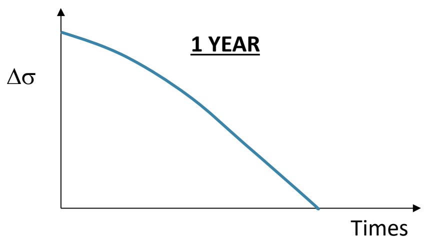
Use ruler and divide the horizontal times axis (Log axis) into n intervals



Level	σ_{max}	σ_{min}	$\Delta\sigma$	N	Occurences
1					N1
2					N2-N1
3					N3-N2
…..					
n					Nn-N(n-1)

Stress history from rainflow count

- The exceedance diagram could also be constructed in terms of stress ranges – which are typically found from rainflow counting results



Exceedance diagrams from a great number of structures are available and can be used in calculations

Clipping

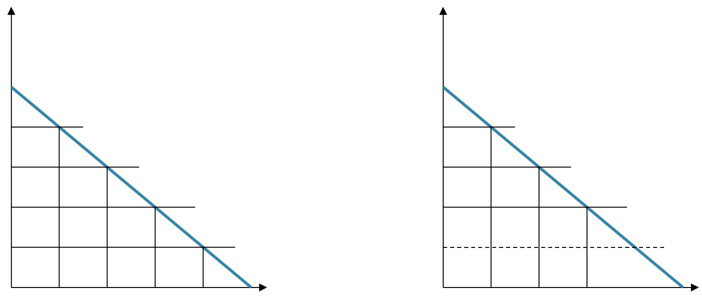
- Clipping is a counting error because a certain level was not exceeded

Truncation

- Truncation is the process to reduce the number of small stress cycles to save on computing time
- Process reconstructs the lower step
- Requires judgement and evaluation of its effect
- The larger the number of stress levels, the less the effect of truncation

Truncation - graphically

Stress levels are the intervals into which the exceedance diagram has been divided into



Manipulation of stress history

- Clipping
- Different periods of severity
- Reasonable number of stress levels [(10-12)x2]

For counting and damage

- Most service histories have variable amplitude
  - Sometimes stochastic of nature (random probability distribution – may be analysed statistically)
- Aspects to consider:
  - Nature of fatigue damage & how it can be related to the load history
  - Damage summation methods
  - Cycle counting techniques to recognise damaging events
  - Crack propagation behaviour
  - How to deal with service load histories
- Fatigue is tendency of materials to fail due to cracks that initiates & propagates

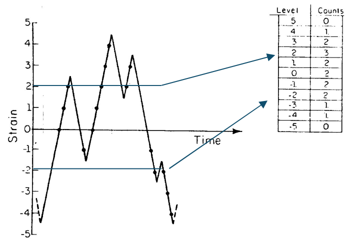
- Definition of fatigue damage:
  - Measurable propagation portion of fatigue
    - Damage is directly related to crack length – it is observable, measurable
    - Inspection intervals to monitor crack growth
  - Initiation phase
    - Mechanisms on microscopic level (dislocations, slip bands, micro-cracks, etc.)
    - Only measurable in highly controlled laboratory environment

  Most damage summing methods during initiation phase empirical of nature

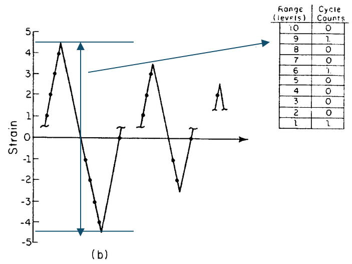
STEP 1: REORDER STRESS HISTORY
To start and end at the peak or valley with the maximum absolute value. If repetitions of max absolute value, start & stop with first peak or valley.

Cycle counting

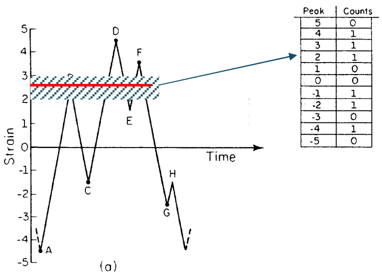
- Purpose of cycle counting techniques
  - Reduce amount of data
  - Ranges or extrema required, and number thereof
- Level-crossing counting

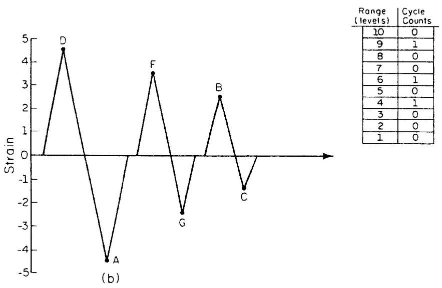


Note that on +'ve value & +'ve slope
Inverse for –'ve value & -'ve slope



Peak-counting



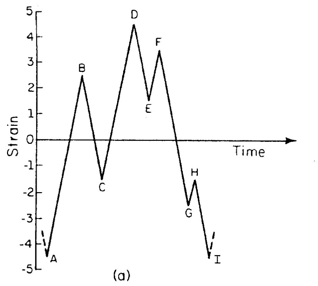


Counts the number of peaks in specific range

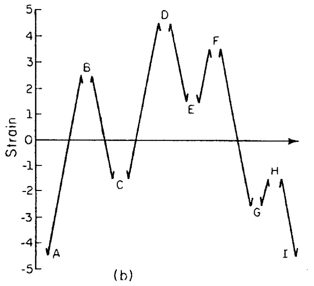
In this case, range is 1

There is only one peak with value 2, that is Point B

Upscaling is done in most algorithms-conservative

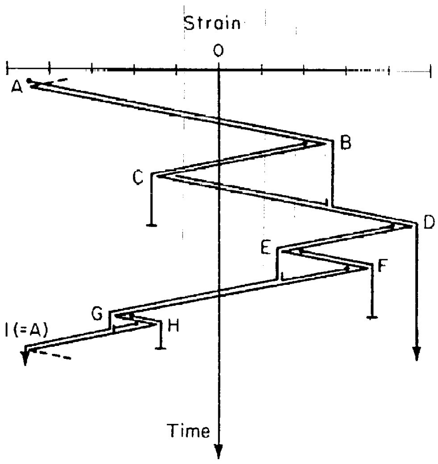


Ranges are counted



Range (levels)	Cycle Counts
10	0
9	0
8	0
7	.5
6	1
5	0
4	.5
3	1
2	.5
1	.5

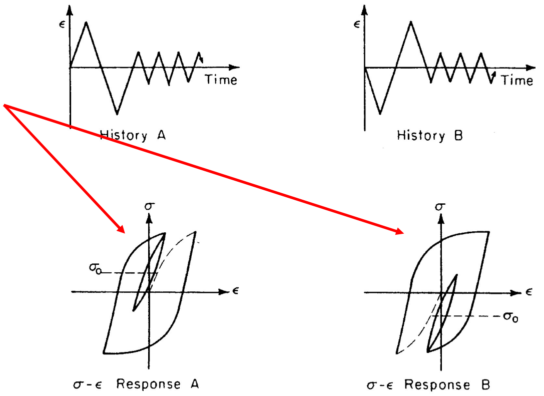
Rainflow counting



- Do live demonstration in class
- Videos:
  - https://youtu.be/rIsy4fDXDwM
  - https://youtu.be/Fvk64Qn9K7E

Sequence effects

- Strain-time histories may yield very different stress-strain responses
- Level-crossing, peak-counting and simple-range counting do not include the sequence effects
- Sequential application of the rainflow counting method and using the from-to values, do provide a means to model sequence effects



Strain life calculations can be applied to model the effect of sequence dependent residual stress

More on rainflow counting

- More info on Rainflow counting
  - A number of rainflow counting techniques are in use
  - If the strain-time history being analyzed begins and ends at the strain value having the largest magnitude, whether it occurs at a peak or a valley, all of the rainflow counting techniques yield identical results
  - Develop the Markov Matrix and find strain amplitudes and mean stress
  - Use the Morrow equation to solve the fatigue life at each strain level

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_f' - \sigma_o}{E}\left[2N_f\right]^b + \epsilon_f'\left[2N_f\right]^c$$

  - Calculate cumulative damage from Palmgren-Miner's rule: $D = \sum \frac{0.5}{N_f} \geq 1$, note, reversals or half cycles are used in the Morrow equation
  - ASTM standard for Rainflow counting in literature

Load Interaction Effects of crack growth

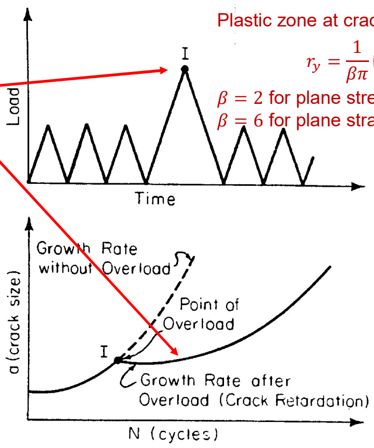
- Crack growth retardation
  - Single overload causes a decrease in crack growth rate
    - If overload is large enough, crack arrest can occur
    - Crack growth retardation remains in effect for a period related to the size of the plastic zone
      - The larger the plastic zone, the longer the crack growth retardation remains in effect
  - Periodic overloads are not always beneficial
    - In low-cycle fatigue (LCF) it may cause crack growth acceleration
- Compressive overloading (underloading)
  - result in tensile residual stress
  - crack growth acceleration occurs

Plastic zone at crack tip:

$$r_y = \frac{1}{\beta\pi}\left(\frac{K}{f_y}\right)^2$$

$\beta = 2$ for plane stress

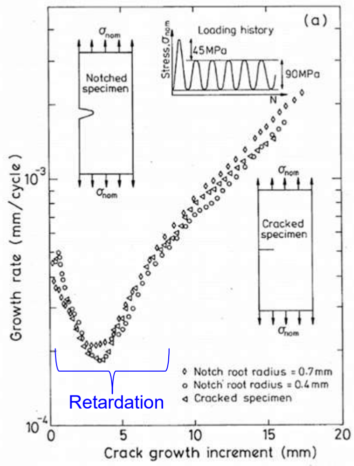
$\beta = 6$ for plane strain



What is the crack growth retardation mechanism?

- Crack-tip blunting
  - Crack tip blunts during overload
    - stress concentration becomes less severe
    - slower crack growth rate
  - Is not in line with practise where it was found that crack growth retardation comes in effect after crack has grown a portion through the plastic zone ***(Delayed retardation)***
- Compressive residual stresses
  - Reduce the effective stress
    - reduction in crack growth rate
  - Does not predict ***delayed retardation***
- Crack closure models
  - Variations in opening tress
    - stress intensity
  - This model does predict ***delayed retardation***

THE NEXT STEP IS TO DEVELOP MODELS FOR CRACK CLOSURE

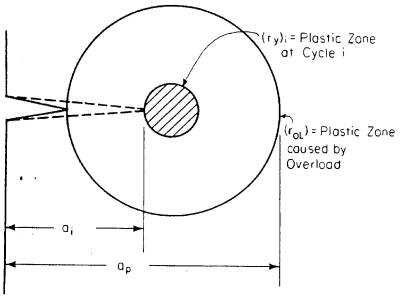


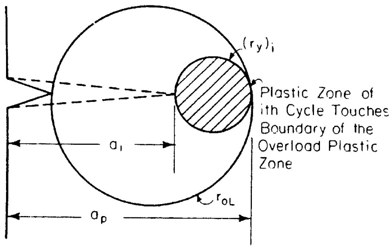
Source: (Shin & Fleck, 1987, p. 386)

Crack-tip plasticity model

If $\Delta K_b \gg \Delta K_{th}$ & plane stress:
- Crack closure

If $\Delta K_b \approx \Delta K_{th}$ & plane strain:
- Strain hardening & residual stress





Crack growth retardation prediction method approaches

- Statistical Methods
  - Use the root mean square stress intensity factor
  - Only applicable to short spectra
  - Do not account for load sequence effects
  - Very restricted application
  - Does not predict crack growth retardation
- Crack closure models
  - Does predict crack growth retardation
  - Must estimate the opening stress for variable loading
  - Must be done cycle for cycle
  - Good correlation has been obtained

Block (repetition) loading

- Use blocks (repetitions) instead of cycle-for-cycle counting
- Considerable savings in time
- Limited to short spectra of loading
- Crack growth per block less than the plastic zone caused by the largest load cycle
    - The block should be short enough that the crack do not propagate to outside the plastic zone during
- Damage is assumed to occur only when the crack is open
    - The crack opening stress must be determined
    - Use change in the positive stress intensity values
        - ❑ Compressive residual stress beneficial

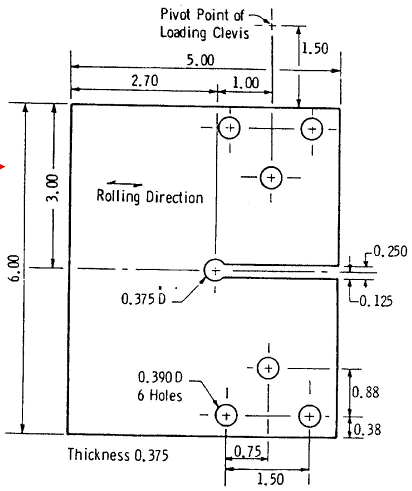
Block (repetition) loading

- Dealing with service histories
    - Sometimes the service load history is unknown
    - A representative load history or loading block may be determined from field tests
        - ❑ Analytical methods also used
    - Fatigue life, or damage may then be calculated from the load blocks

Method: SAE Cumulative Damage Test program

Cumulative damage test program

- SAE Cumulative Damage Test Program
  - The component used in the study
  - Two steels were used:
    - ❑ ManTen
      - Yield strength = 80 ksi = 552 MPa
    - ❑ RQC-100
      - Yield strength = 120 ksi = 827 MPa
  - Different loadings were used



Cumulative damage test program

- A series of tests were done to determine
  - ❑ Baseline material strain-life and crack growth data
  - ❑ Constant amplitude component load-life data
  - ❑ Variable amplitude component data
- The following analysis techniques were used to predict lives
  - ❑ Rainflow counting was used to find ranges
  - ❑ Miner's rule was used for damage summation
  - ❑ The life analysis was done using
    - Stress-life approach and the fatigue strength reduction factor K_f
    - Load-life curves
    - Local strain approach
      - Neuber analysis using K_f
      - Finite element analysis results
      - Assumption of elastic strain behaviour
      - Load-strain calibration curves using strain gauge measurements
    - Analysis were made ignoring and considering mean stresses
  - ❑ Techniques were also used to condense load histories
  - ❑ No analysis was made of crack propagation lives

Cumulative damage test program

- Results
  - There was not a significant difference in the predictions made by any method that **used a reasonable estimate of notch root stress-strain behaviour**
  - Good predictions were made using the Neuber approach that tended to be slightly conservative
  - There was not a large difference between predictions which included and excluded mean stresses
  - Predictions made using the simple **stress-life approach** showed correlation which was as good as those predicted by more complicated techniques
  - Another study showed that the following method predicted very good propagation lives
    - Use FEA to determine crack opening levels
    - **Rainflow counting** + Linear Elastic Fracture Mechanics (LEFM)

Conclusion

- Miner's linear damage rule provides reasonable life estimates
- Most effective cycle counting procedures relate damaging events to the stress-strain response of the material (Like Rainflow counting)
- Repeated block loading analysis techniques may be applied to save time
- Application of large overloads **may** cause crack growth retardation