Stress-life

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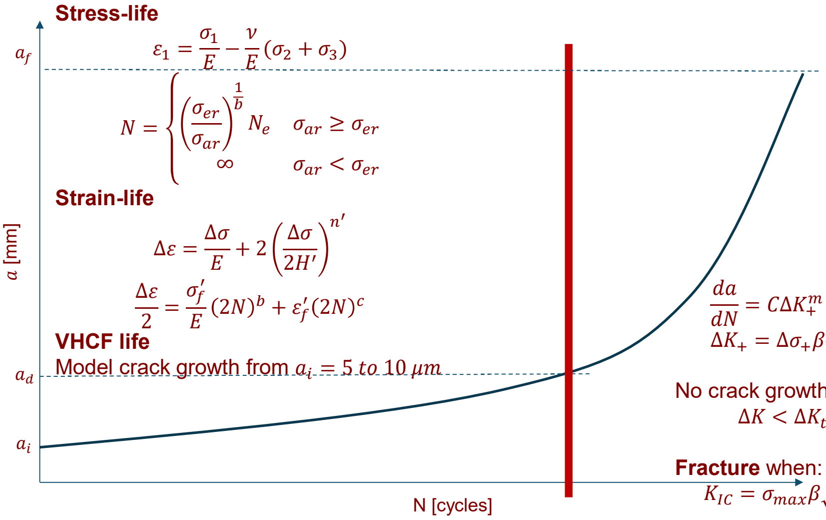
Variable stress on metal part

- Metals and alloys
  - Variable loads causes dislocation movements
  - This eventually forms persistent slip bands that nucleate short cracks
  - This damage is cumulative & does not recover when the material is rested
  - Macroscopic cracks will begin to form
  - Crack will reach a critical size
    - Plastic collapse
    - Fracture

Fatigue and Fracture



Stress-life

$$\varepsilon_1 = \frac{\sigma_1}{E} - \frac{\nu}{E}(\sigma_2 + \sigma_3)$$

$$N = \begin{cases} \left(\frac{\sigma_{er}}{\sigma_{ar}}\right)^{\frac{1}{b}} N_e & \sigma_{ar} \geq \sigma_{er} \\ \infty & \sigma_{ar} < \sigma_{er} \end{cases}$$

Strain-life

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2H'}\right)^{n'}$$

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_f'}{E}(2N)^b + \varepsilon_f'(2N)^c$$

VHCF life

Model crack growth from $a_i = 5\ to\ 10\ \mu m$

$$\frac{da}{dN} = C\Delta K_+^m$$

$$\Delta K_+ = \Delta\sigma_+ \beta\sqrt{\pi a}$$

No crack growth when:

$$\Delta K < \Delta K_{th}$$

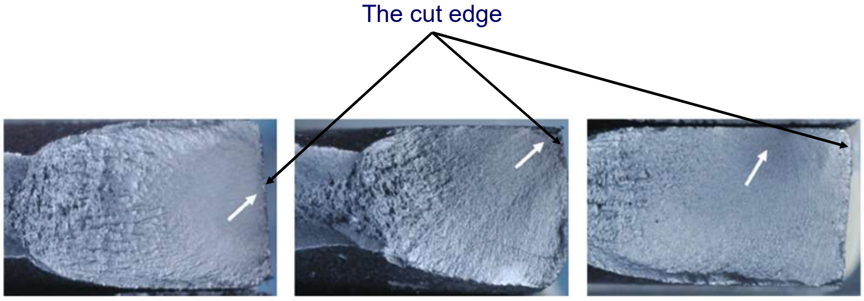
Fracture when:

$$K_{IC} = \sigma_{max}\beta\sqrt{\pi a_f}$$

Plastic collapse when:

$$\sigma_{max} \geq \sigma_o$$

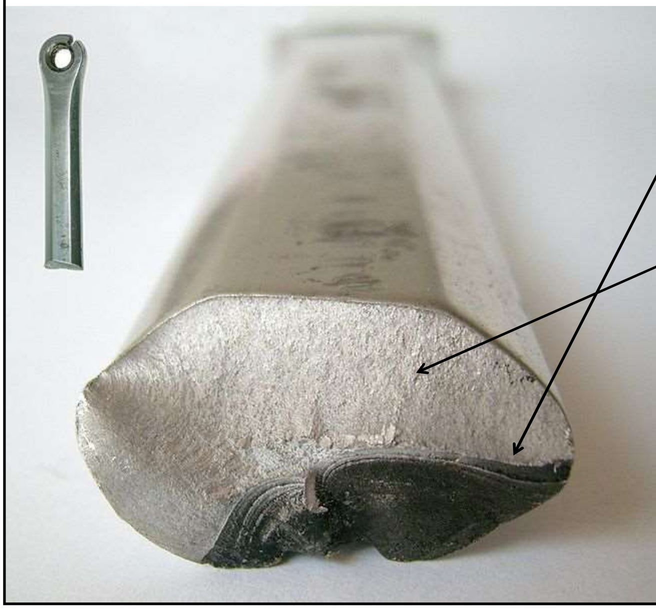
Crack Initiation Points at cuts



At cut edge **At corner** **At plate surface**

Diekhoff, P., Hensel, J., Nitschke-Pagel, T. & Dilger, K., 2020. Investigation on fatigue strength of cut edges produced by various cutting methods for high-strength steels. *Welding in the World,* Volume 64, pp. 545-562.

Failure surfaces



Fracture of an Aluminium crack arm

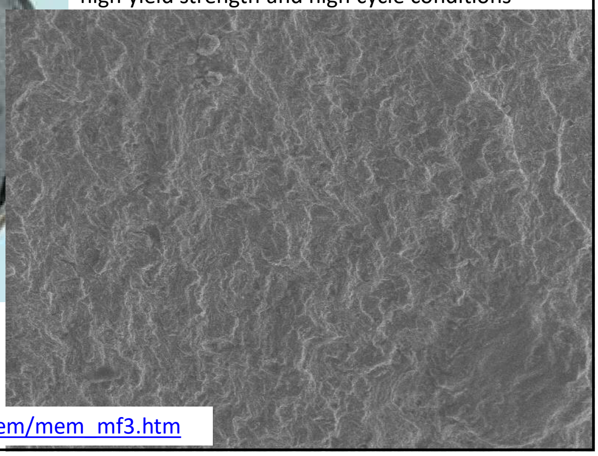
Area of crack propagation. 'beach' marks due to crack propagation is clearly shown

Area of fracture – bright

http://en.wikipedia.org/wiki/File:Pedalarm_Bruch.jpg

Failure of bolt

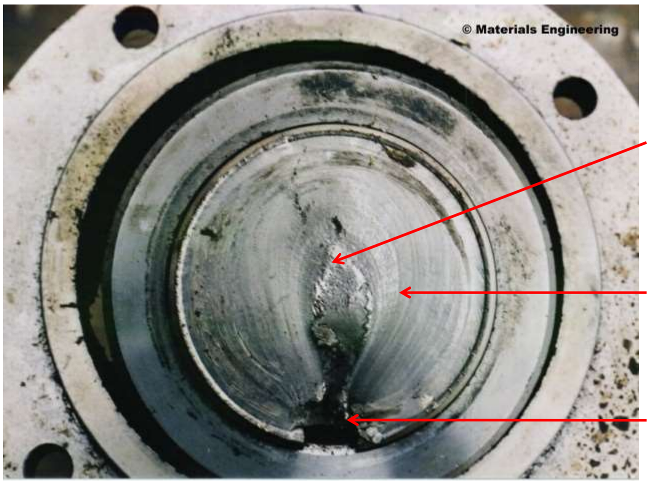
High tensile steel bolt. Failed under low stress high cycle conditions. The SEM image of the fatigued surface is found to have no striations due to the high yield strength and high cycle conditions



Crack initiated here – see beach marks caused by propagation

http://materials.open.ac.uk/mem/mem_mf3.htm

LHD shaft failure

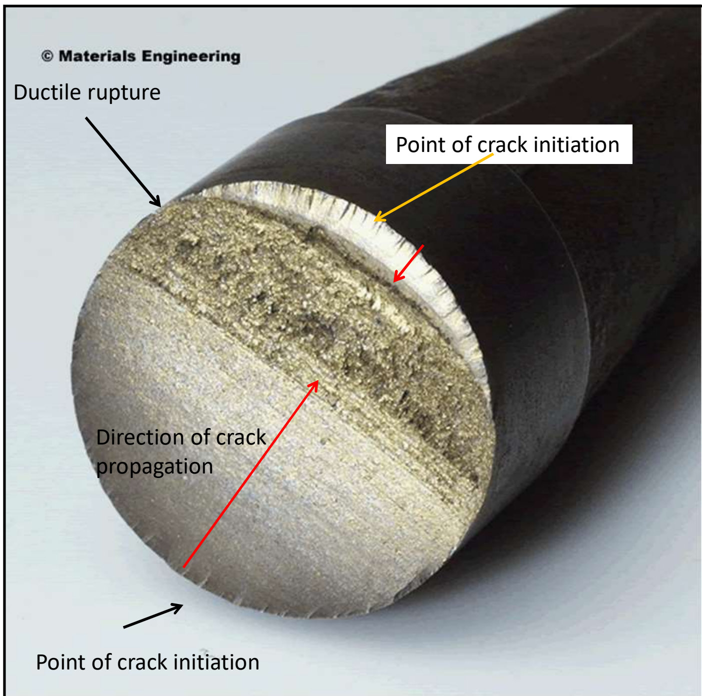


Fracture occurred at this rougher area

Crack propagated in circumferential direction due to reversed bending

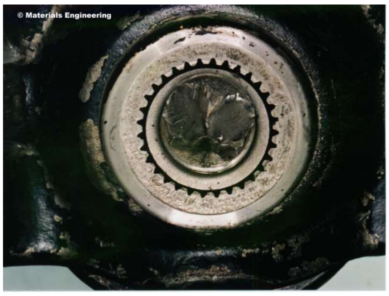
Crack initiated at keyway corners

http://materials.open.ac.uk/mem/mem_mf8.htm



Stub axle loaded in reverse bending
Crack initiated at 8 o'clock and 2 o'clock
Propagated towards each other

Reversed torsional fatigue failure of shaft



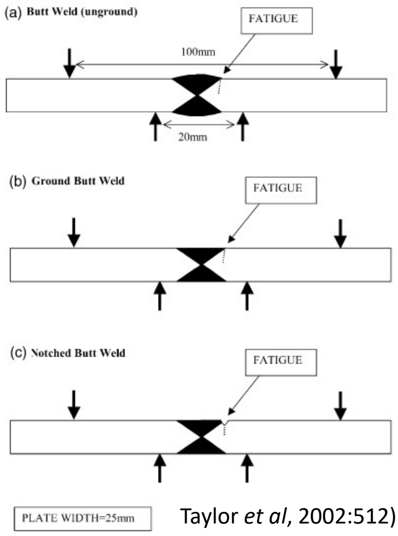
http://materials.open.ac.uk/mem/mem_mf7.htm

1. Two cracks initiated at circumferential recess adjacent to end of splines
2. Cracks propagated into cross section in helical paths
   1. Cycles of forces in opposing directions (forward and reverse travelling of vehicle),
      each crack follows opposing helices
3. Crack propagation at 90° angle to shaft axis due to bending

Notches - Introduction

- Failure criteria on static load
  - Maximum principal stress
  - Von Mises
  - Tresca, maximum shear stress, maximum normal stress
- Stress-life approach
  - Cannot account for load sequence events
- Strain-life approach
  - Account for notch root plasticity
  - Account for the influence of load sequence effects on local mean and residual stresses
- Fracture Mechanics
  - Account for crack growth at a notch

Stress concentrations



Taylor *et al*, 2002:512)

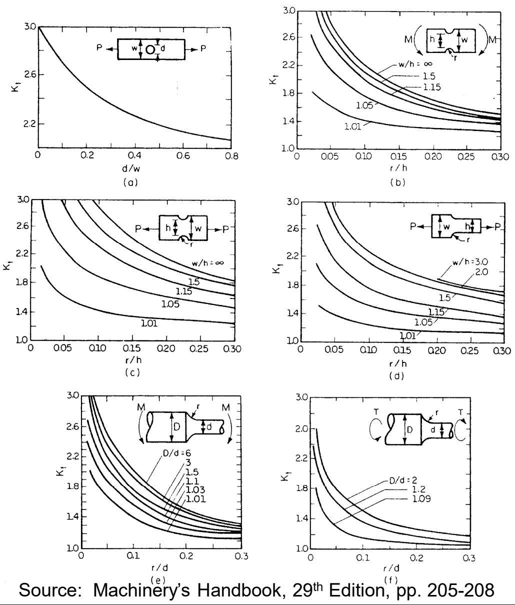
- Geometrical or micro-structural discontinuities
- Result in maximum local stress, σ

$$K_t = \frac{\sigma}{S}$$

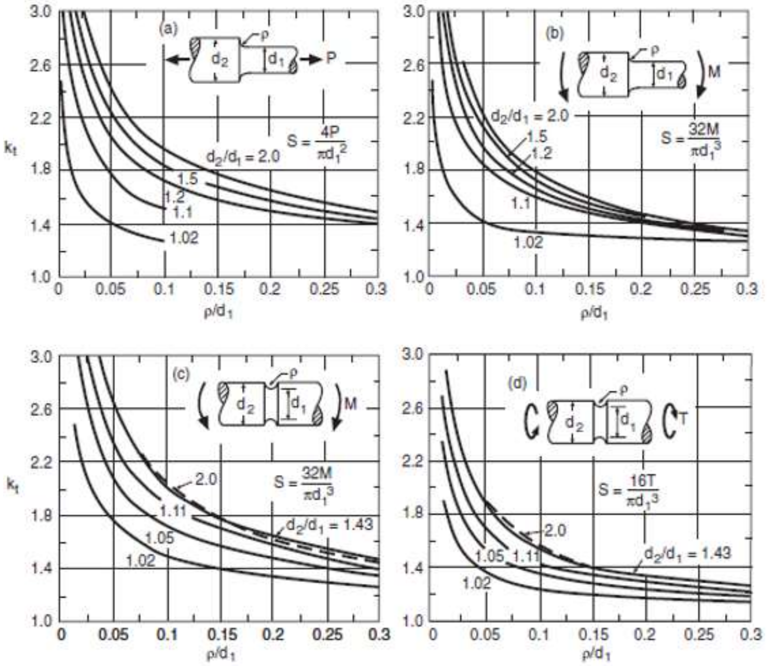
- Local stress higher than nominal stress S
- Ratio is theoretical stress concentration factor K_t

Notches - Stress-life approach

- K_t depends on geometry and mode of loading
- Make sure where you should calculate the stress, in the reduced, thicker, thinner, etc. part



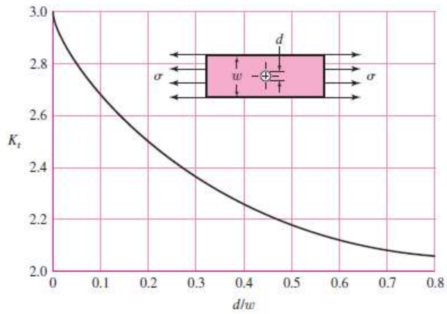
Source: Machinery's Handbook, 29th Edition, pp. 205-208



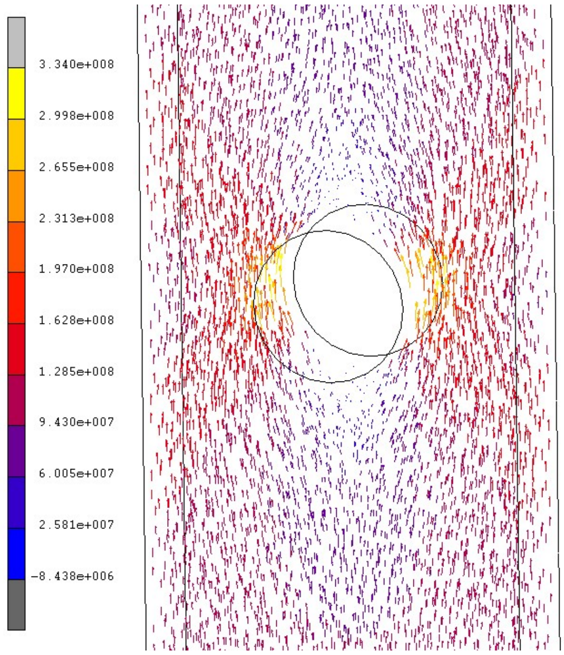
Source: (Dowling, 2013, p. 892)

Thin plate with centre hole

- Nominal stress S to use in the fatigue calculation is the average stress over the nett-section at the hole:
$$S = \frac{F}{(W-d)t}$$
- The theoretical stress concentration factor is the point stress σ divided by the nominal stress:
$$K_t = k_t = \frac{\sigma}{S} \geq 1$$
- Always make sure where to calculate the average stress to be used with the stress concentration diagram



Source: Budynas & Nisbet. 2008. *Shigley's Mechanical Engineering Design*, 8th Ed, pp. 106. McGraw-Hill.



In this example, a 50mm wide, 25 mm thick and 400 mm long section was loaded with a nominal stress of 100 MPa. The component has a hole with diameter 20 mm. Ratio d/W = 0.4. The stress tensor is shown to the left.

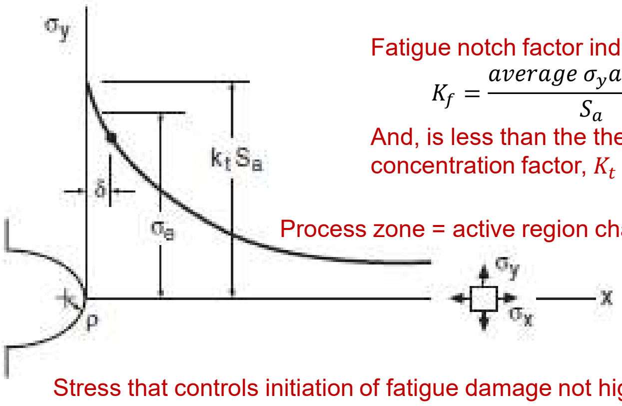
According to the applicable table Kt ~ 2.3. The stress analysis shows that the stress is 3.340 x the nominal stress. However, modeling area ratios the stress concentration is 3.340 x 30/50 = 2.00. What is correct?

Notches - Stress-life approach with fatigue notch factor

Use the fatigue notch factor K_f - a reduction factor at long life (10^6 to 10^7 cycles)

$$K_f = \frac{\sigma_{ar}}{S_{ar}} = \frac{\sigma_e}{S_e}$$

Stress distribution at a notch – notch fatigue factor, K_f



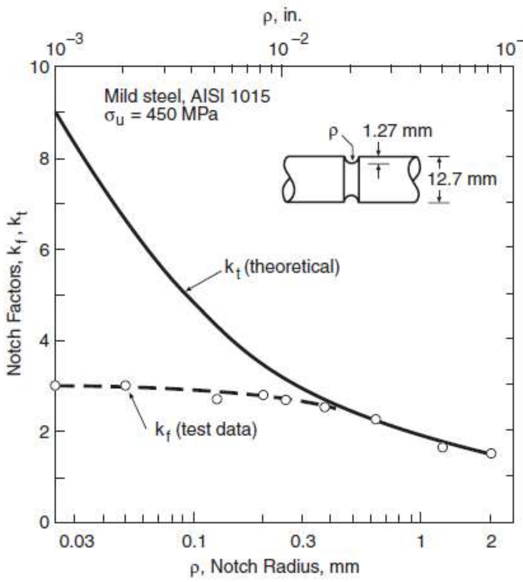
Fatigue notch factor indicated at δ:

$$K_f = \frac{average\ \sigma_y at\ x = \delta}{S_a} = \frac{\sigma_e}{S_e}$$

And, is less than the theoretical stress concentration factor, K_t

Process zone = active region characterised by δ

Stress that controls initiation of fatigue damage not highest at $x = 0$, but, at depth δ



Why is K_f less at smaller notch radius?

<u>Stress gradient</u> at notch
For small notch radius ρ, stress gradient with increasing x away from the notch is more abrupt

<u>Weakest link argument:</u>
Small volume subject to high stress where the presence of voids, inclusions, or other microscopic stress raisers are less

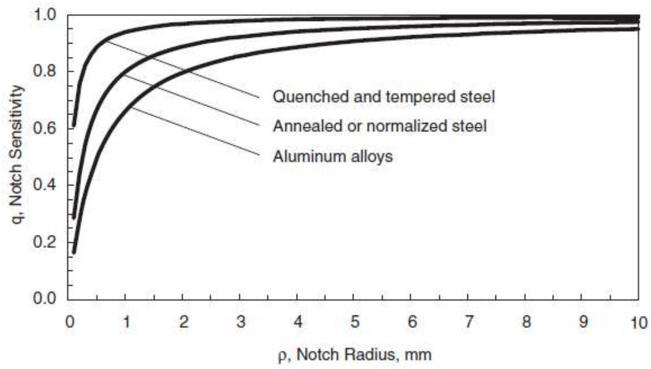
<u>Crack growth effect</u>
Cracks initiate that relieve stress

<u>Reversed yielding effect:</u>
Due to yielding the actual stress amplitude in the notch is less than $K_t S_a$
Not sufficient as there is no yielding in engineering materials around 10^6 to 10^7 cycles.

Notch sensitivity and fatigue notch factor

Notch sensitivity q:

$$q = \frac{K_f - 1}{K_t - 1} = \frac{1}{1 + \frac{\alpha}{\rho}}$$



Estimates of α

Estimate:

$$\alpha = \left[\frac{300}{f_u[ksi]}\right]^{1.8} \times 10^{-3}\ in$$

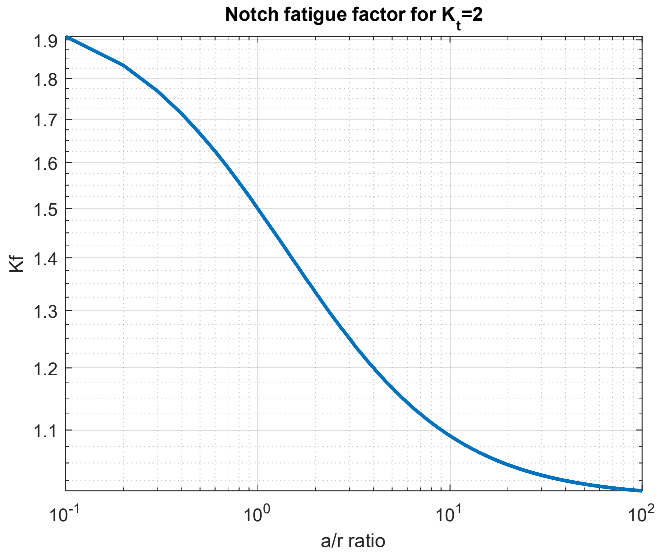
Peterson for steel:

$$\log \alpha = 2.654 \times 10^{-7}\sigma_u^2 - 1.309 \times 10^{-3}\sigma_u + 0.01103$$
$$\alpha\ [in\ mm] = 10^{\log \alpha}\ \ (345 \le \sigma_u \le 2\,070\ MPa)$$

For typical materials

α [mm]	Material
0.51	Aluminium alloys
0.25	Annealed or normalized low-carbon steels (BHN ~ 170)
0.064	Quenched and tempered steels (BHN ~ 360)
0.0254	Highly hardened steels (BHN ~ 600)

Notch fatigue factor for $K_t = 2$



Notch sensitivity in terms of Neuber constant

Neuber:

$$q = \frac{K_f - 1}{K_t - 1} = \frac{1}{1 + \sqrt{\frac{\beta}{\rho}}}$$

For steel:

$$\log\beta = -1.079 \times 10^{-9}\sigma_u^3 + 2.740 \times 10^{-6}\sigma_u^2 - 3.740 \times 10^{-3}\sigma_u$$

$$\beta\ [in\ mm] = 10^{\log\beta}\ \ (345 \leq \sigma_u \leq 1\,725\ MPa]$$

For Aluminium:

$$\log\beta = -9.402 \times 10^{-9}\sigma_u^3 + 1.422 \times 10^{-5}\sigma_u^2 - 8.249 \times 10^{-3}\sigma_u$$

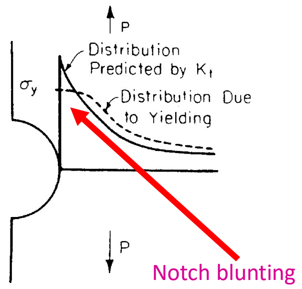
$$\beta\ [in\ mm] = 10^{\log\beta}$$

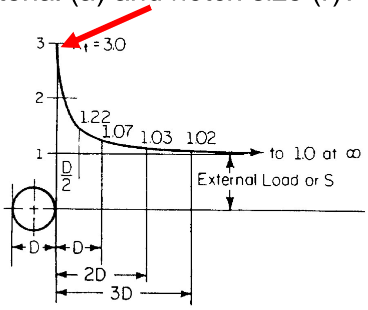
Notches - Stress-life approach

- Q: What does the notch fatigue factor do as function of notch radius?
- Q: Why are hard steels more notch sensitive than soft steels? (Hint: compare values for a).

Notches - Stress-life approach - Blunting

- Q: Why is K_f dependent on material (a) and notch size (r)?

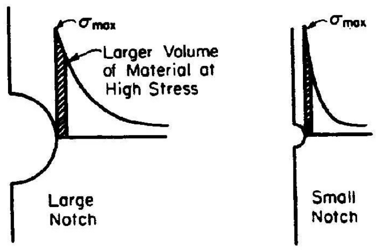




- MATERIAL: Yielding at the notch root
  - Peak stresses as predicted by K_t are never attained in soft materials – due to yielding at the notch root
  - In high strength materials, full effect of K_t is attained

Notches - Stress-life approach

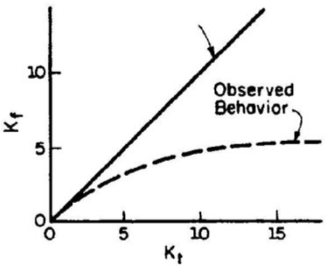
- NOTCH SIZE:
  - Volume effect



Notches - Stress-life approach

- There is a limiting value between 5 and 6 for K_f

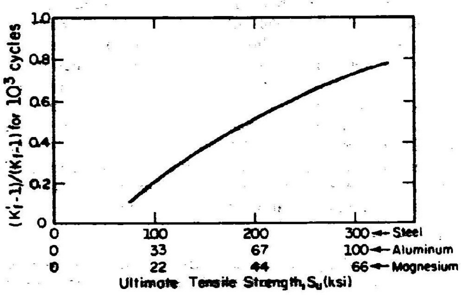
$$q = \frac{K_f - 1}{K_t - 1}$$
$$K_f = q(K_t - 1) + 1$$



  - Reasons:
    - Blunting effect
    - Initiation-propagation effects in sharp notches. **Initiation life is small in sharp notches**. Therefore, total life is dependent on crack propagation.

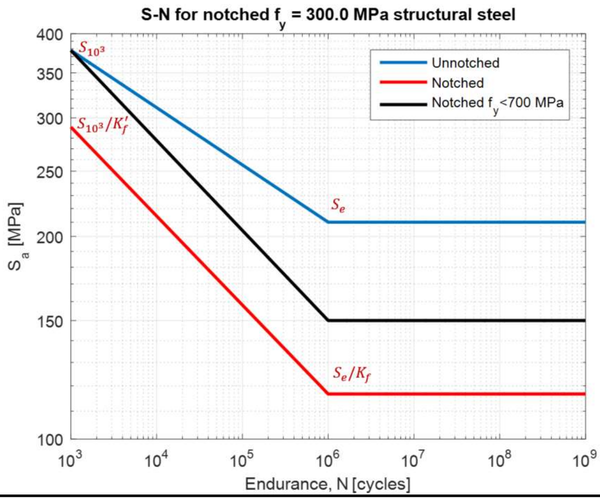
Notches - Stress-life approach

- Q: How do we use K_f in fatigue life analysis?
  - Adjust the S-N curve
    - The fatigue notch factor is a function of loading (Blunting)
    - Fatigue notch factor at 1000 cycles are defined as K_f'
    - An empirical relationship exists between K_f' and K_f
    - This is given by:



Notches - Stress-life approach

This is now used to find the new S-N curve:



No yielding at the notch, $K'_f = K_t$

Local yielding: $K'_f = \frac{\sigma_o}{S_a}$

Full yielding: $K'_f = 1$

Notches - Stress-life approach

- Stress-life approach
  - Best suited for high cycle fatigue (HCF)
  - Notch strains must be predominantly elastic
  - Loading essentially constant in amplitude
  - Does not account for inelastic behaviour at the notch
  - Cannot properly account for changes in notch mean or residual stresses
  - Account for load sequence events in an empirical manner
  - Especially useful for long lives where surface finish, manufacturing processes, temperature, etc. have a large effect

Problem

300WA structural steel has the following material properties:

E = 206 GPa, f_y = 300MPa,

f_{ut} =450MPa

Assume a notch fatigue factor of $K_f = 1.7$

What is the endurance limit for $P_{survival} = 0.5$?

How many cycles to failure at:

1. S_a= 200MPa
2. S_a= 300MPa

Solution

For this problem, the endurance limit is equal to:

$$S_e = \frac{S_{ut}}{2k_f} = \frac{450}{2 \times 1.7} = 132.4\ MPa$$

This is also the completely reversed stress amplitude at 1 million cycles

To solve the remaining part of the question we need to calculate the S-N curve. At 1000 cycles the following applies:

$$S_{1000} = 0.9S_{ut}/K_f' = C10^{3b}$$

$$S_{1000}^m \cdot 1000 = 132.4^m \cdot 10^6$$

$$\left(\frac{0.9S_{ut}}{\frac{S_{ut}}{3.4}}\right)^m = 10^3$$

$$m = \frac{3}{\log_{10}(0.9 \times 3.4)} = 6.1$$

From the graph: $(K_f'-1)/(K_f - 1 = 0$, therefore, $K_f' = 1$

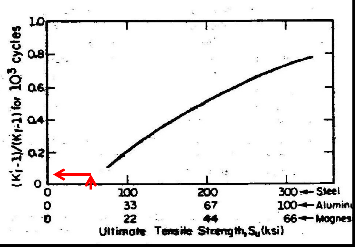
At the endurance limit we have:

$$132.4 = C10^{6b}$$

Dividing the two equations give:

$$\frac{132.4}{0.9 \times 450} = 10^{3b}$$

$$b = \frac{1}{3}[log(0.3269)] = -0.1619$$



Solution

Dividing the two equations give:

$$\frac{132.4}{0.9 \times 450} = 10^{3b}$$
$$b = \frac{1}{3}[log(0.3269)]$$
$$= -0.1619$$

The coefficient C is then:

$$C = \frac{0.9S_{ut}}{10^{3\times -0.1619}}$$
$$= 1{,}238.9$$

The S-N curve is then given by:

$$S = CN^b$$
$$N = \left(\frac{S}{C}\right)^{\frac{1}{b}}$$

For a stress amplitude of 200 MPa we have:

$$N = \left[\frac{200}{1238.9}\right]^{\frac{1}{-0.1619}}$$
$$= 78{,}199\ cycle$$

Some issues

- Ignores true stress-strain behavior
- Treat all strains as elastic
- Endurance or fatigue limit S_e exists for certain materials – primarily body centered cubic (BCC) steels
  - Due to interstitial elements (carbon nitrogen) in iron, which pins dislocations
    - This prevents slip mechanism which causes the formation of micro-cracks
  - Endurance limit may disappear due to
    - Periodic overloads
    - Corrosive environments
    - High temperatures
    - Machining processes, etc.
  - Most non-ferrous alloys have no endurance limit – S-N line has continuous slope
- The S-N approach should not be used to estimate lives below 1,000 cycles

Mean stress correction

- Haigh (Master) diagrams expensive to determine
  - Because one is needed for the required stress combinations
- Equation must be found to determine fatigue behaviour at an equivalent alternating stress with R = -1 (which is also completely reversed)
- For this case (R = -1), the S-N diagram could be used
- Approach used was to correct S_e on alternating stress axis to S_y, S_{ut} or σ_f (fracture stress)

Mean stress correction

Soderberg: $\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_y} = 1$

Morrow: $\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\tilde{\sigma}_{fB}} = 1$

Goodman: $\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_u} = 1$

Gerber: $\frac{\sigma_a}{\sigma_{ar}} + \left(\frac{\sigma_m}{\sigma_u}\right)^2 = 1$

- What is the effect on the equivalent completely reversed stress σ_{ar} if there are tensile mean stresses in the signal?
- What is the effect on the equivalent completely reversed stress σ_{ar} be if there are compressive mean stresses in the signal?
- Conclusion: Compressive mean stresses are beneficial and allow large alternating stresses

Smith, Watson & Topper (SWT) mean stress correction

Smith, Watson & Topper:

$$\sigma_{ar} = \sqrt{\sigma_{max}\sigma_a}$$
$$= \sigma_{max}\sqrt{\frac{1-R}{2}}$$

Advantage of not relying on any material constant

Single curve on a plot of σ_a/σ_{ar} vs σ_m/σ_{ar}

Walker

Walker:

$$\sigma_{ar} = \sigma_{max}^{1-\gamma}\sigma_a^{\gamma} \qquad (\sigma_{max} > 0)$$
$$= \sigma_{max}\left(\frac{1-R}{2}\right)^{\gamma} \qquad (\sigma_{max} > 0)$$

Equivalent to SWT with $\gamma = 0.5$

Single curve on a plot of σ_a/σ_{ar} vs σ_m/σ_{ar}

Dowling give following estimation of γ:

$$\gamma = -0.000200\sigma_u + 0.8818 \quad (\sigma_u in\ MPa)$$

So, which mean stress correction shall we use?

- Neither Goodman nor Gerber very accurate
- Goodman overly conservative
- Gerber is non-conservative
- Morrow reasonably accurate, but, use fracture strength that is not always known
- Morrow relationship with σ_f' fits data well, but not for aluminium and non-ferrous alloys
- Soderberg very conservative and seldom used
- Actual test data tend to fall between Goodman and Gerber curves
- SWT good choice for aluminium
- Walker best choice where γ is available
- For most design situations, R<1 (small mean stress in rel. to alt. stress) – little difference in theories – **<u>Use Goodman, it is based on ultimate tensile strength that is available</u>**

$$\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_u} = 1$$

$$\sigma_{ar} = \frac{\sigma_a}{1 - \frac{\sigma_m}{\sigma_u}}$$

Example

Component undergoes cyclic stress with:

$$\sigma_{max} = 770MPa$$
$$\sigma_{min} = 70MPa$$

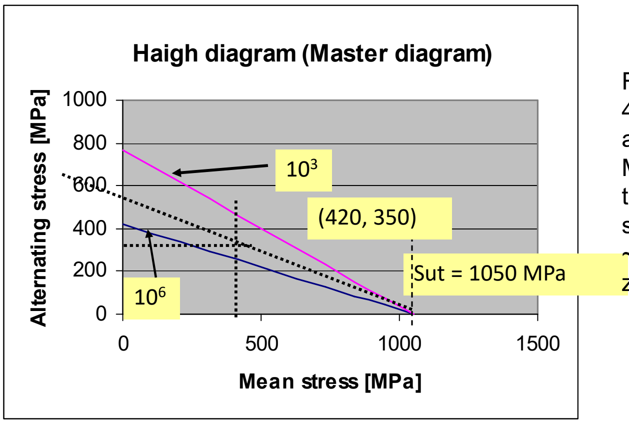
Material is steel with S_{ut} = 1,050 MPa and S_e = 420 MPa. The fully reversed stress at S_{1000} = 770 MPa.

How many cycles can be loaded on the component until fatigue crack initiation? That is, what is the fatigue life of the component?

Solution

- The mean stress is (770+70)/2 = 420MPa
- The amplitude of the stress is (770-70)/2 = 350 MPa

Solution – use Haigh diagram



From the mean of 420 MPa and the amplitude of 350 MPa it is clear that the alternating stress shows to be ~550 MPa for a zero mean.

Solution

- From Goodman:

The alternating stress calculated from this equation can now be used on the S-N curve to calculate the life (in cycles)

$$\frac{\sigma_a}{S_a} + \frac{\sigma_m}{S_{ut}} = 1$$
$$\frac{\sigma_a}{S_a} = 1 - \frac{\sigma_m}{S_{ut}}$$
$$S_a = \frac{\sigma_a}{1 - \frac{\sigma_m}{S_{ut}}}$$
$$= \frac{350}{1 - {420}/{1050}}$$
$$= 583\ MPa$$

S-N curve slope and endurance

- Could be done graphically
- Analytically:

$$S_1^m N_1 = S_2^m N_2$$
$$\left(\frac{S_1}{S_2}\right)^m = \frac{N_2}{N_1}$$
$$m = \frac{\log_{10}\frac{N_2}{N_1}}{\log_{10}\frac{S_1}{S_2}}$$
$$= \frac{\log_{10}\frac{10^6}{10^3}}{\log_{10}\frac{770}{420}}$$
$$= 11.8567$$

The endurance at 583 MPa is:

$$583^m N = 430^m \times 10^6$$
$$N = \left(\frac{430}{583}\right)^m \times 10^6$$
$$= 27\,074\ cycles$$

Remarks on the Solution:

- Compressive mean stresses beneficial and allow larger alternating stresses, or smaller S_a for constant σ_a. Different for notched specimens due to residual stress.
- Bannantine *et all* (1990:10): mean shear stress has no effect on life when added to alternating shear stress. Not true for notched specimens.

Modifying factors

General trend of modification factors is to have less effect at short lives
Usually specified for the endurance limit (subscript r means completely reversed – zero mean stress, b means rotating bending):

$$S_{er} = S'_{erb} C_{load} C_{size} C_{surf} C_{rel} C_T$$

$$S'_{erb} = \frac{m_e \sigma_u}{K_f}$$

At 1 000 cycles:

$$S_{ar,10^3} = S'_{arb,10^3} C_{load} C_{rel} C_T$$

$$S'_{arb,10^3} = \frac{m' \sigma_u}{K'_f}$$

Modification factors are empirical models
- Great care must be taken when extrapolating these
- Conduct tests under specific conditions

Factors have more pronounced influence as the strength of base steel increases
- For low strength carbon steels – effects have little effect
- Because residual stresses relax out easier out of materials with low yield strengths

Modifying factors: Dowling equivalent

In Dowling, the factors are:

Type of loading: $m_t = C_{load}$
Size: $m_d = C_{size}$
Surface finish: $m_s = C_{surf}$
Other effects: $m_o = C_T$ $for\ temperature, corrosion, etc$

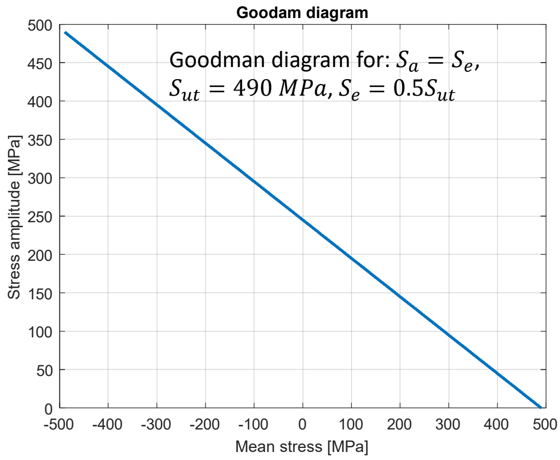
Goodman Diagram

- Goodman diagram
  - Plot of σ_a vs σ_m at on the endurance line

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = 1$$

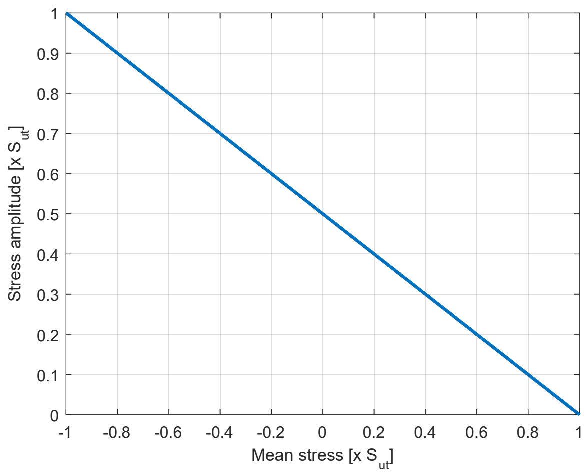
$$\sigma_a = S_e\left(1 - \frac{\sigma_m}{S_{ut}}\right)$$

- S_e replaced by S_a



The diagram can be constructed for any endurance

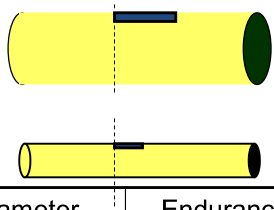
Goodman diagram normalized to S_{ut} at endurance for $S_e = 0.5S_{ut}$



Smith diagram = self study

Modification factors – Size effects

- Correlates with thin layer of surface material subjected to ≥95% of the maximum surface stress
- Large component has less steep stress gradient = larger volume of material subjected to high stress
- Greater probability of initiating fatigue crack in large components
- Test results (Bannatine *et all*, 1990:12):



Diameter [in]	Endurance limit [ksi]
0.3	33.0
1.5	27.6
6.75	17.3

$$C_{size} = \begin{cases} 1.0 & \text{if } d \leq 8\text{mm} \\ 1.189 d^{-0.097} & \text{if } 8\text{mm} \leq d \leq 250\text{mm} \end{cases}$$

Size modification factor - Thickness

The size factor, C_{size}, is given by the following for bending and torsion (Budynas & Nisbett, 2012, p. 280):

$$C_{size} = 1.51 d^{-0.157}$$

Where:

d is the section thickness [m]

Modifying factors – loading effects

- A conservative estimate for endurance limits between axial and bending stress is:

$S_{e,axial} = 0.70 S_{e,bending}$

$C_{load} = 0.7\ if\ S - N\ curve\ is\ from\ bending\ tests$

  - If the S-N curve was constructed from a bending test, the load factor for a specimen subjected to axial loading will be approximately 0.7
  - However, if the S-N curve used was constructed from axial tests, the load factor for a specimen loaded in bending will be $\frac{1}{0.7} = 1.43$.
- Torsion: $\tau_{er}(torsion) = 0.577 S_{e,bending}$

Modifying factor – load effect

- For torsion:

$$\sigma_1 = \tau, \sigma_2 = 0, \sigma_3 = -\tau$$

$$\text{Von Mises}: 2S_y^2 = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2$$

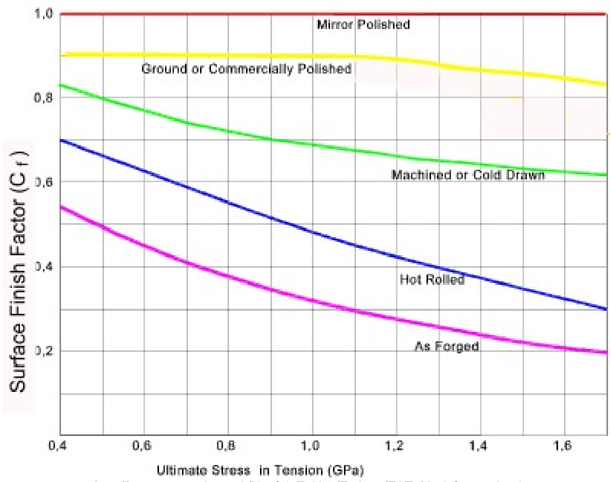
$$= \tau^2 + \tau^2 + 4\tau^2$$

$$= 6\tau^2$$

$$\therefore \tau_e(torsion) = \frac{1}{\sqrt{3}} S_y = 0.577 S_e(bending)$$

Modifying factors – surface finish

- Scratches, pits, machining marks, etc. ads stress concentrations to ones already present due to component geometry
- Uniform fine-grained materials (high strength steel) more aversely affected by surface finish than coarse-grained material (cast iron)



http://www.roymech.co.uk/Useful_Tables/Fatigue/FAT_Mod_factors.html

Surface finish modifying factors - Calculated

- Surface finish modifying factors calculated according to Roymech (http://www.roymech.co.uk/Useful_Tables/Fatigue/FAT_Mod_factors.html)

$$C_f = a \times S_{ut}^b$$

- With

Surface finish	a [MPa]	b
Ground	1.58	-0.085
Machined or Cold Drawn	4.51	-0.265
Hot Rolled	57.7	-0.718
As Forged	272	-0.995

Modifying factors – surface treatment

- Can have significant effect (crack initiates at free surface)
- In plating, thermal and mechanical (welding, milling, pressing, etc) treatment, effect on fatigue life due primarily to residual stresses
- Should residual stresses result – pre-stressing or pre-setting should be done to produce compressive residual stresses at free surface

Presetting
Initial overload of component
Only favorable for loads in the direction of overload
Should not be used in cases of fully reversed loading

Plating

- Chrome and Nickel plating of steels can cause up to 60% reduction in endurance limits
  - High tensile stresses are generated by plating process
  - To alleviate residual stress problem:
    - ❑ Nitride part before plating
    - ❑ Shot peen part before or after plating (Best to peen after plating)
    - ❑ Bake or anneal the part after plating
- Corrosion resistance offered by plating can more than offset the reduction in fatigue strength seen in non-corrosive environment
- Plating with cadmium and zinc appear to have no effect on fatigue strength
- Electroplating can cause hydrogen embrittlement

Modifying factor - thermal

- Diffusion processes such as carburizing and nitriding beneficial for fatigue strength
  - Produces higher strength material on surface
  - Causes volumetric changes that produce residual compressive stresses
- Flame and induction hardening
  - Cause phase transformation which in turn cause volumetric expansion
  - If localized to surface – compressive residual stresses result that is beneficial for fatigue strength

Modifying factors - thermal

- Hot rolling and forging
  - Cause surface decarburization
  - Loss of carbon atoms from surface causes lower strength and may produce residual stresses
  - Both factors are detrimental to fatigue strength
- Manufacturing processes such as grinding, welding, flame cutting etc.
  - Can set up detrimental residual tensile stresses
  - Shot peening effective to undo damage caused by these processes

Modifying factor - temperature

- Endurance limits of steels increase at low temperatures (watch out for brittleness)
- Endurance limit for steels disappears at high temperatures due to mobilization of dislocations
- For $T > T_{melt}/2$ creep becomes important
  - Stress-life approach no longer applicable.
- Annealing happens at high temperatures that may remove beneficial residual compressive stresses

Modifying factor - Thermal

- According to Roymech:

$$C_T = \begin{matrix} 1.0\ for\ T \leq 450\ ℃ \\ 1 - 5.8^{-3}(T - 450), for\ 450 < T \leq 550\ ℃ \end{matrix}$$

Source: http://www.roymech.co.uk/Useful_Tables/Fatigue/FAT_Mod_factors.html

Modifying factor - Reliability

Reliability $1 - p_f$	C_r
0.5	1
0.9	0.897
0.95	0.868
0.99	0.814
0.999	0.753
0.9999	0.702
0.99999	0.659
0.999999	0.620

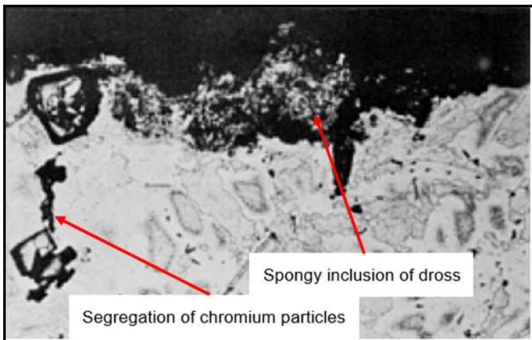
http://www.roymech.co.uk/Useful_Tables/Fatigue/FAT_Mod_factors.html

Modifying factor - mechanical

- Cold work processes – rolling & shot peening
  - Produce compressive residual stresses
    - Gives the greatest improvement in fatigue life
  - Work-hardens the material
  - Rolling cause deep stress layer (bolts, etc)
  - Shot peening gives (compressive stress = $0.5S_y$) layer of ~1 mm

Modifying factor - mechanical

- Shot peening:
  - ❑ leaves dimpled surface: hone or polish part after shot peening
  - ❑ Undo deleterious effects caused by chrome and nickel plating, decarburization, corrosion, grinding, etc.
  - ❑ Steels with $F_y \leq$ 550MPa seldom cold rolled or shot peened (Easy to introduce plastic strains that wipe out residual stresses)
  - ❑ Surfaces can be overpeened! Subsurface failures may occur!



Source: (ASM Vol. 9, 1999, p. 758)

Defect	Fatigue strength reduction factor
None	1.00
Dross	0.54
Micro shrinkage	0.73
Macro shrinkage	0.50
Chunky graphite	0.75
Anomalies	0.83
Source: (Satef, 2023)	

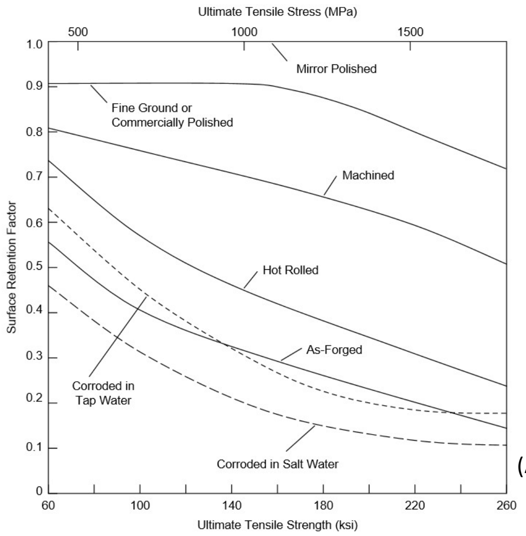
Misrun and cold shuts (cold laps)

- See handout notes

Modifying factor - environment

- Corrosion-fatigue
  - Corrosive environment attacks surface – produces an oxide film
  - Oxide film serve as protective layer
  - Cyclic loading causes localized cracking of layer – exposing fresh material to corrosive environment
  - Cause localized pitting – stress concentrations
  - To improve corrosion-fatigue resistance:
    - Paint and plating (chrome, nickel, cadmium and zinc)
      - Nickel reduce fatigue strength in non-corrosive environment, but improve fatigue strength in corrosive environment
    - Nitriding, shot peening, cold rolling, etc.
- Loading frequency
  - Similar data at various frequency in non-corrosive environment
  - Corrosion-fatigue are greatly influenced by loading frequency

Corrosion reduction factors



(ASM International, 2008:16)

Example – to do unprepared in class

Component undergoes axial cyclic stress as follows, which was obtained by Rainflow counting of the stress signal for one repetition OF 3 months.

Stress amplitude [MPa]	Stress mean [MPa]	Number of cycles/repetition
100	200	10000
50	0	5000
100	100	20000
200	-100	2000

Material is steel with S_{ut} = 1,050 MPa with hardness 350 BHN. The theoretical stress concentration factor at a notch on the part is $K_t = 2$. The notch radius is $r = 4\ mm$. The surface finish is machined. The shaft has a radius of 100 mm and operates at temperature $\mathrm{T} = 500\ °C$.

How many repetitions of loading can be loaded on the component for a 1 % probability of fatigue crack initiation? That is, what is the fatigue life of the component for probability of survival 99 %?

Use Goodman, Walker & SWT mean stress compensation and compare.

Multi-axial fatigue

- See Investmech Fatigue (Multi-axial fatigue) P R0.0

Effect of inclusions on fatigue

- Murakami model
  - Puff, R. & Barbieri, R. 2014. Effect of non-metallic inclusions on the fatigue strength of helical spring wire. *Engineering Failure Analysis 44, pp. 441-454*.

Murakami model

- Include the inclusion size in the calculation (Puff&Barbieri, 2014:445):

$$\sigma_w = \frac{1.56 \cdot (H_v + 120)}{\left(\sqrt{A_{incl}}\right)^{\frac{1}{6}}} \cdot \left[\frac{1 - R_w}{2}\right]^{\alpha}$$

$$\alpha = 0.226 + H_v \times 10^{-4}$$

$$A_{incl} = \frac{\pi}{4} D_{max}^2$$

Where,
D_{max} is the maximum dimension of the inclusion section **in [μm]**
R_w is the load ration

- For torsion, multiply with 0.67
- Source: Puff, R. & Barbieri, R. 2014. Effect of non-metallic inclusions on the fatigue strength of helical spring wire. *Engineering Failure Analysis 44, pp. 441-454*.
  - ScienceDirect.com

For example

In an application where $H_v = 572$, and $D_{max} = 144\ \mu m$ and the load ration $R_w = -0.867$ the fatigue endurance strength (at 10^6 cycles) with the inclusion is:

$$\alpha = 0.226 + H_v \times 10^{-4}$$
$$= 0.226 + 572 \times 10^{-4}$$
$$= 0.283$$
$$A_{incl} = \frac{\pi}{4} D_{max}^2 = \frac{\pi}{4}(144)^2 = 1.629 \times 10^4$$
$$\sigma_w = \frac{1.56 \cdot (H_v + 120)}{\left(\sqrt{A_{incl}}\right)^{\frac{1}{6}}} \cdot \left[\frac{1 - R_w}{2}\right]^{\alpha}$$
$$= \frac{1.56 \cdot (572 + 120)}{\left(\sqrt{1.629 \times 10^4}\right)^{\frac{1}{6}}} \cdot \left[\frac{1 - (-0.867)}{2}\right]^{0.283}$$
$$= 472\ MPa$$

Note, in the calculation of the inclusion area, the characteristic dimension of the inclusion must be in microns!

Conclusions

- Endurance limit only exist in plain carbon and low-alloy steels
- Following factors will reduce the endurance limit
  - Tensile mean stress, large section size, rough surface finish, chrome and nickel plating (except in corrosive environment), decarburization due to forging and hot rolling, severe grinding
- Following factors tend to increase the endurance limit:
  - Nitriding, flame and induction hardening, carburization, shot peening, cold rolling
  - Chrome and nickel plating for materials in a corrosive environment

References

- http://materials.open.ac.uk/mem/mem_mf.htm
- TAYLOR, D., BARRETT, N. & LUCANO, G. 2002. Some new methods for predicting fatigue in welded joints. *International Journal of Fatigue 24 (2002) 509-518.*