Strain-life analysis

Dr. Michiel Heyns Pr.Eng.
T: +27 12 664-7604
C: +27 82 445-0510
mheyns@investmech.com

Objective

- Understand the strain life method
- Apply the relevant equations for
  - Zero and non-zero means
  - Multiaxial stress
- Apply strain-based method for fatigue life estimates

Background

- Response of the material in critical locations (notches) is strain or deformation dependent
  - Plastic deformation occur at notch, material hardens/softens, etc.
  - Material under elastic stress away from the notch
- At low loads – stress and strain are linearly related
  - In this range load-controlled and strain-controlled test results are equivalent
- Early fatigue showed damage is dependent on plastic deformation or strain
- In strain-life – local yielding (plastic strain) or deformation is directly measured and quantified
  - Stress-life does not account for plastic strain
- Strain-life can also be used where there is little plasticity & long lives
  - Comprehensive method

Background

- Nominal loads normally remain elastic
  - Stress concentrations often cause local plastic strains
- Due to constraint imposed by elastically stressed material surrounding the plastic zone, deformation at a notch root is considered strain-controlled
- Crack growth is not explicitly accounted for in the strain-life method
  - Failure is assumed to occur when the ‘equally stressed volume of material’ fails
  - Strain-life methods consider ‘initiation’ life estimates

Background

- Local strain-life approach
  - Best method to model short fatigue lives
  - Usefull method of evaluating the fatigue life of a notched component
  - Best method to model sequence effect of occasional severe events
- American Society for Testing and Materials (ASTM) and the Society of Automotive Engineers (SAE) have recommended procedures and practices for conducting strain-controlled tests and using these data to predict fatigue lives

Stress- and strain-based approaches to fatigue

Stress-based approach	**Strain-based approach**
Use nominal stress	Use local stress and strains (local yielding)
Elastic concentration factors	Cyclic stress-strain relationship at notch
Nominal stress vs life (S-N)	Strain vs life $(\varepsilon - N)$
Good for long lives	Good for short and long lives
Mean nominal stress	More rational & accurate handling of mean stress effect by employing the local mean stress at the notch
No specific analysis of crack propagation (growth)	No specific analysis of crack propagation (growth)
Can not model the sequence effect of severe events	Best method to model sequence effects of severe events

Fatigue life prediction method

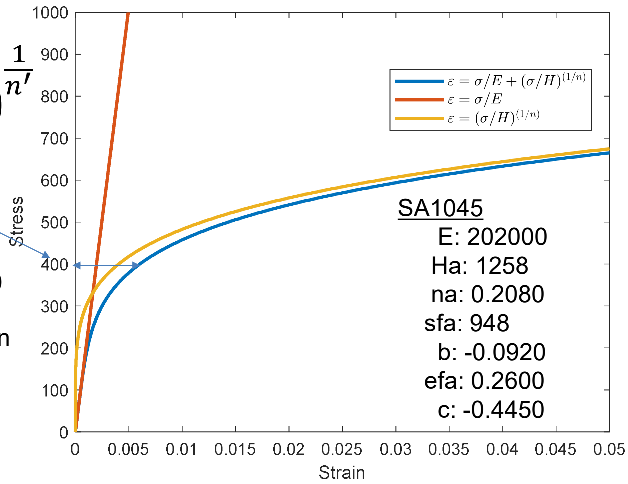
1. Obtain material properties
   - Literature, lab tests
2. Stress-strain history at critical local location
3. Identify damaging events (cycle counting)
4. Calculate zero-mean-strain fatigue life
5. Do mean stress correction
6. Damage summation (Palmgren-Miner rule)

Ramberg-Osgood stress-strain

Formula

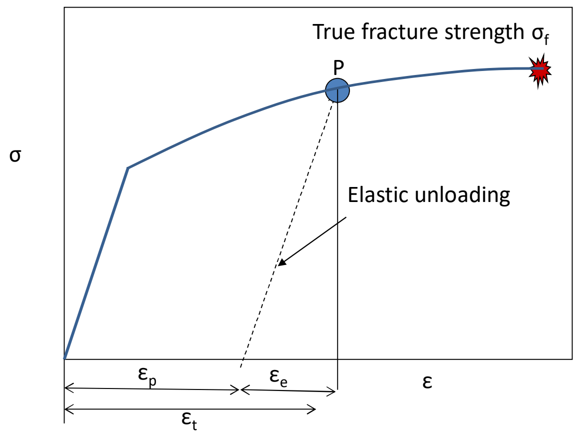
$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}}$$

The accent (n' and H') indicates that stable hysteresis stress-strain curve parameters are used



Material behaviour – Stress-strain

TRUE STRESS VS. STRAIN



Linear elastic strain (ε_e): portion of strain recovered upon unloading

Plastic strain (ε_p) portion that cannot be recovered

Material behavior – monotonic (load increased to failure) stress-strain behaviour

Engineering stress:

$$\sigma = \frac{P}{A_i}$$

Engineering strain:

$$\varepsilon = \frac{\ell - \ell_i}{\ell_i} = \frac{\Delta\ell}{\ell_i}$$

A_i original area

P applied load

ℓ instantaneous length $= \ell + \Delta\ell$

ℓ_i original length

True stress:

$$\tilde{\sigma} = \frac{P}{A}$$

True strain:

$$\tilde{\varepsilon} = \int_{\ell_o}^{\ell} \frac{1}{\ell} d\ell = \ln\frac{\ell}{\ell_i}$$
$$= \ln(1 + \varepsilon)$$

A instantaneous area

Monotonic means the specimen is tensioned until failure. No cycles.

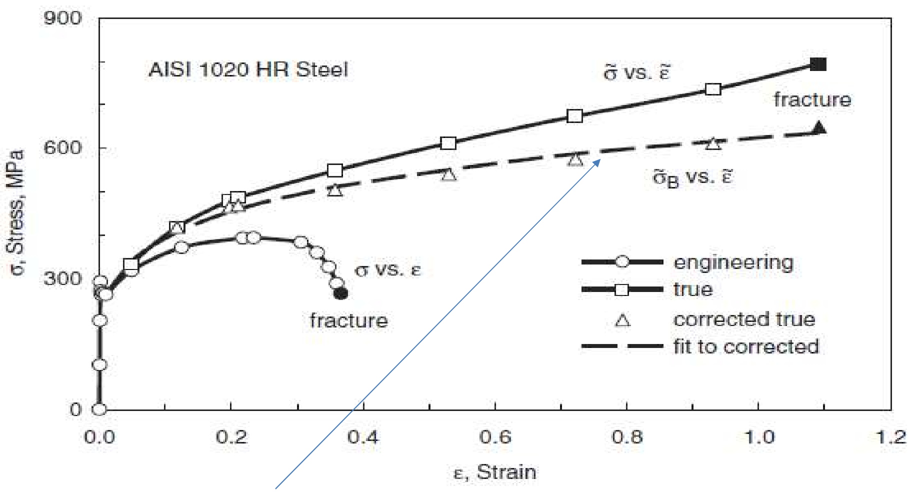
Material behaviour – monotonic stress-strain behaviour

Constant volume assumption

$$A_i L_i = AL$$
$$\frac{A_i}{A} = \frac{L}{L_i} = \frac{L_i + \Delta L}{L_i} = 1 + \varepsilon$$
$$\tilde{\sigma} = \sigma(1 + \varepsilon)$$
$$\tilde{\varepsilon} = \ln\left(\frac{A_i}{A}\right)$$

Constant volume assumption and round sections

$$\tilde{\varepsilon} = \ln\left(\frac{A_i}{A}\right) = \ln\left(\frac{\frac{\pi}{4} d_i^2}{\frac{\pi}{4} d^2}\right) = 2\ln\left(\frac{d_i}{d}\right)$$



Bridgman correction for steels due to tensile hoop stress that forms during necking

$$\tilde{\sigma}_B = B\tilde{\sigma}$$
$$B = \begin{cases} 1 & \tilde{\varepsilon} < 0.12 \\ 0.0648x^3 + 0.0461x^2 - 0.205x + 0.825 & 0.12 \le \tilde{\varepsilon} \le 3 \\ not\ defined\ yet & \tilde{\varepsilon} > 3 \end{cases}$$
$$x = \log_{10} \tilde{\varepsilon}$$

We need stress-strain relationships

Elastic, perfectly plastic

$$\sigma = \begin{cases} E\varepsilon & \sigma \leq \sigma_o \\ \sigma_o & \sigma \geq \sigma_o \end{cases}$$

Elastic, linear-hardening

$$\sigma = \begin{cases} E\varepsilon & \sigma \leq \sigma_o \\ (1-\delta)\sigma_o + \delta E\varepsilon & \sigma \geq \sigma_o \end{cases}$$

Elastic, power-hardening

$$\varepsilon = \begin{cases} \frac{\sigma}{E} & \sigma \leq \sigma_o \\ \left(\frac{\sigma}{H_1}\right)^{\frac{1}{n_1}} & \sigma \geq \sigma_o \end{cases}$$

An estimate for yield strength

$$\sigma_o = E\left(\frac{H_1}{E}\right)^{\frac{1}{1-n_1}}$$

Ramberg-Osgood

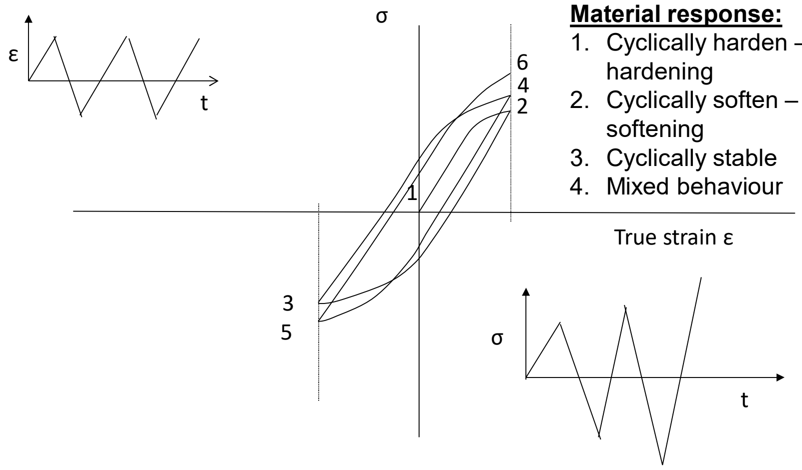
$$\varepsilon = \varepsilon_e + \varepsilon_p$$

$$= \frac{\sigma}{E} + \left(\frac{\sigma}{H}\right)^{\frac{1}{n}}$$

Yield strength estimate from Ramberg-Osgood

$$\sigma_o = H(0.002)^n$$

Material behaviour – Hysteresis loops



Material response:

1. Cyclically harden – strain hardening
2. Cyclically soften – strain softening
3. Cyclically stable
4. Mixed behaviour

Material behaviour – strain hardening

- Soft material
  - Initial dislocation density low
  - Density increases due to cyclic plastic straining
  - Significant strain hardening results
- Hard material
  - Subsequent strain cycling causes rearrangement of dislocations
  - This offers less resistance to deformations
  - Material cyclically softens

$\frac{\sigma_u}{\sigma_o} > 1.4$ material will cyclically harden

$\frac{\sigma_u}{\sigma_o} < 1.2$ material will cyclically soften

σ_u is the monotonic ultimate strength. σ_o is the 0.2% offset yield strength

Strain hardening

$$\varepsilon_t = \frac{\sigma}{E} + \left(\frac{\sigma}{H}\right)^{\frac{1}{n}}$$

- For the strain hardening exponent $n > 0.2$, the material will cyclically harden
- For $n < 0.10$, the material will cyclically soften
- Transient behaviour (hardening or softening) occurs only during the early fatigue life
  - After this the material achieves a cyclically stable condition

Stable hysteresis curves

- Can be determined using the following methods:
  - **Companion samples** – a series of companion samples are tested at various strain levels until the hysteresis loops become stabilized
  - The tips of the loops are then connected to form the stable cyclic stress-strain curve
  - Incremental step test
    - One specimen subjected to series of blocks of gradually increasing and decreasing strain amplitudes
    - After few blocks the material stabilizes
    - Cyclic stress-strain curve then determined by connecting the tips of the stabilized hysteresis loops
- Massing's hypothesis states that the stabilized hysteresis loop may be obtained **by doubling the cyclic stress-strain curve**

$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}}$$

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2H'}\right)^{\frac{1}{n'}}$$

Cyclically stable stress - plastic strain for stable hysteresis curve

Ramberg-Osgood (cyclically stable)

$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}}$$

Elastic part: $\varepsilon_{ae} = \frac{\sigma_a}{E}$

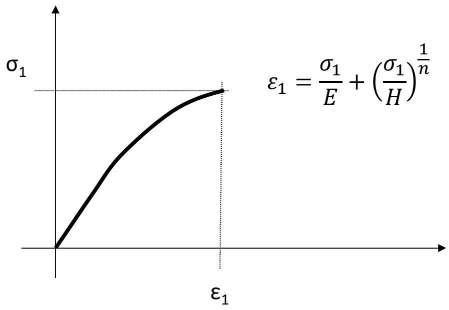
Plastic part: $\varepsilon_{ap} = \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}}$, or, $\sigma_a = H'\varepsilon_{ap}^{n'}$

Where:

σ_a	The cyclically stable stress amplitude [MPa]
ε_{ap}	The cyclically stable plastic strain amplitude
E	Modulus of elasticity [MPa]
H'	Cyclically stable strength coefficient [MPa]
n'	Cyclically stable strain hardening exponent

Stabilized cyclic stress-plastic strain curve

- Cyclic stress-strain curve is used for initial application of strain, normally from zero to a given stress or strain
- Use monotonic stress-strain curve (of you have the coefficients and constants)



$$\varepsilon_1 = \frac{\sigma_1}{E} + \left(\frac{\sigma_1}{H}\right)^{\frac{1}{n}}$$

Cyclically stable hysteresis curve

- Use the cyclically stable hysteresis curve equation for all successive strain reversals

$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}}$$

- Solve the equation using absolute values for stress or strain
  - Apply a -1 or +1 to indicate the direction of the change

Monotonic vs cyclically stable values

Monotonic parameters	Cyclically stable parameters
n	n'
H	H'
These are used to define the stress-strain curve	These are used in the subsequent cyclic loading
Parameters determined from a monotonic test – one reversal (half cycle)	Determined from the stable hysteresis loop
$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}}$	

Strain life & mean stress compensation

- Calculate the zero-mean-stress-equivalent fatigue life, N^* from strain-life equation

$$\varepsilon_a = \frac{\sigma_f'}{E}(2N^*)^b + \varepsilon_f'(2N^*)^c$$

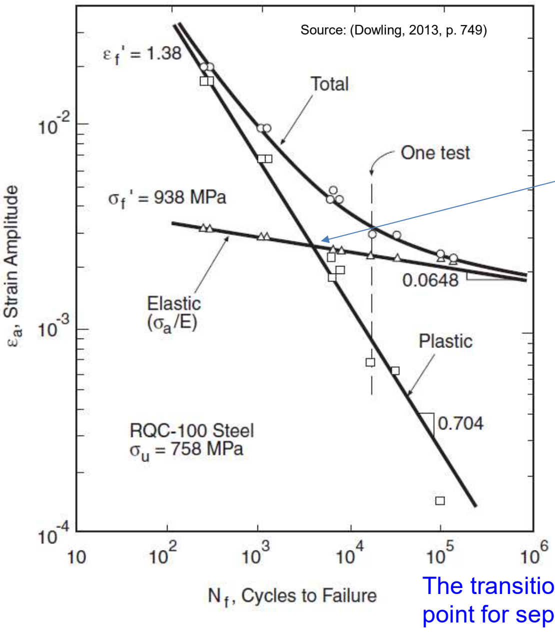
Calculate the mean stress-compensated endurance (fatigue life), N_f, as follows

$$N_{mi}^* = N_f\left(\frac{\sigma_a}{f(\sigma_a, \sigma_m)}\right)^{\frac{1}{b}}$$

$$N_f = N_{mi}^*\left(\frac{\sigma_a}{f(\sigma_a, \sigma_m)}\right)^{-\frac{1}{b}}$$

Where:

N_f	Fatigue life of non-zero mean stress loaded part
N_{mi}^*	Fatigue life for zero mean stress calculated from this equation
$f(\sigma_a, \sigma_m)$	Function that calculates the equivalent completely reversed stress amplitude



Strain life curve

At the transition fatigue life/strength

$$\frac{\sigma_f'}{E}(2N_t)^b = \varepsilon_f'(2N_t)^c$$

$$\frac{(2N_t)^b}{(2N_t)^c} = \frac{\varepsilon_f' E}{\sigma_f'}$$

$$(2N_t)^{b-c} = \frac{\varepsilon_f' E}{\sigma_f'}$$

$$2N_t = \left(\frac{\varepsilon_f' E}{\sigma_f'}\right)^{\frac{1}{b-c}}$$

$$N_t = \frac{1}{2}\left(\frac{\varepsilon_f' E}{\sigma_f'}\right)^{\frac{1}{b-c}}$$

$$= \frac{1}{2}\left(\frac{\sigma_f'}{\varepsilon_f' E}\right)^{\frac{1}{c-b}}$$

The transition fatigue life is the most logical point for separating low-cycle and high-cycle fatigue

Mean-stress compensation models

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Morrow

$$f(\sigma_a, \sigma_m) = \sigma_{ar} = \frac{\sigma_a}{1 - \frac{\sigma_m}{\sigma_f'}}$$

Works well for steels
Inaccurate for Al-alloys, can be improved by taking: $\sigma_f' = \tilde{\sigma}_{fB}$

$$N_f = N_{mi}^*\left(1 - \frac{\sigma_m}{\sigma_f'}\right)^{-\frac{1}{b}}$$

Smith, Watson & Topper (SWT)

$$\sigma_{max}\varepsilon_a = \sigma_{ar}\varepsilon_{ar}$$

$$= \frac{\left(\sigma_f'\right)^2}{E}\left(2N_f\right)^{2b} + \sigma_f'\varepsilon_f'\left(2N_f\right)^{b+c}$$

Acceptable for wide range of materials
Accurate for steels
Accurate for Al-alloys

Walker

$$f(\sigma_a, \sigma_{max}) = \sigma_{ar} = \sigma_{max}^{1-\gamma}\sigma_a^{\gamma} \qquad (\sigma_{max} > 0)$$

$$= \sigma_{max}\left(\frac{1-R}{2}\right)^{\gamma} \qquad (\sigma_{max} > 0)$$

Most accurate of all methods

Needs γ, on estimate is

$\gamma = -0.000200\sigma_u + 0.8818 \quad (\sigma_u \text{ in } MPa)$

$$N_f = N_w^*\left(\frac{\sigma_a}{\sigma_{max}}\right)^{\frac{\gamma-1}{b}}$$

$$= N_w^*\left(\frac{1-R}{2}\right)^{\frac{\gamma-1}{b}}$$

Modified Morrow

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_f' - \sigma_0}{E}(2N_f)^b + \varepsilon_f'(2N_f)^c$$

Test results show good correlation with this formula

Example 2

The cyclic stress-strain and strain-life parameters for a steel is:

E=30 x 10^3 ksi; K' = 154 ksi, σ'_f = 133ksi, ε'_f = 0.26, n' = 0.202, b = -0.095,

c=-0.47

Determine the life for the histories shown on the next slide (use Morrow).

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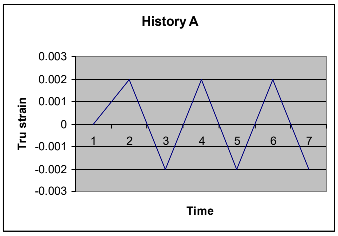
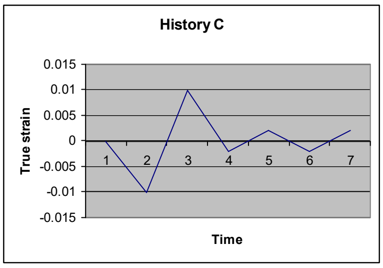
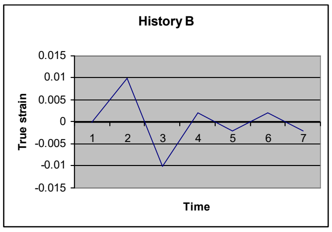
Example 2

- Histories in strainlife.xls.







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Solution 2

- Do in front of class in excel file strainlife.xls

For IIW students just discuss the results in the Excel sheet:
Strain life example – Sequence effect.xls

Notches - Strain-life approach

- Accounts for notch plasticity
- Strain life = f(strain-life history, specimen strain-life data or fatigue properties)
- Sequence dependent
  - Therefore, look at
    - Constant amplitude
    - Variable amplitude
    - Sequence effects
    - Mean stress effects
- To apply, notch root stresses and strains must be known
    - Strain gauge measurements
    - Finite Element Analysis (FEA)
    - Nominal values Local stresses and strains (Normally the least expensive to do - Not always!. Can you give example?.)

Notches - Strain-life approach

- Q: What happens at the notch root for high stress loading?
    - Nominal stresses increase. S and K_t remains constant until yielding begins
    - Upon yielding, the local stress and the local strain are no longer linearly related ➔ they are no longer related by K_t
    - Different stress and strain concentration factors are now implemented:

$$K_\sigma = \frac{\sigma}{S} \; for \; \sigma \le f_y$$

$$K_\varepsilon = \frac{\varepsilon}{e} \; for \; \sigma \ge \mathrm{f}_y$$

Notches - Strain-life approach

- Neuber derived the following relationship between the local stress (σ), the local strain (ε), the nominal stress (S) and the nominal strain (e):

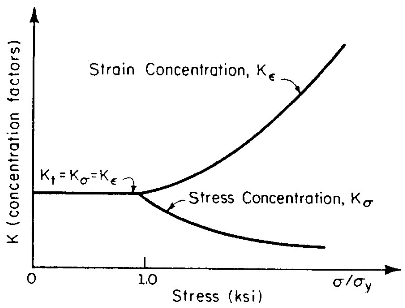
$$K_t = \sqrt{K_\sigma K_\varepsilon}$$
$$K_t^2 = \frac{\sigma\varepsilon}{Se}$$
$$K_t^2 Se = \sigma\varepsilon$$

Neuber's rule:
The geometric mean of the stress and strain concentration factors remain equal to k_t

- This method was only tested for one geometry, but, is assumed to hold for most other notch geometries
- Three versions are used
  - Nominally elastic behaviour
  - Limited yielding
  - Seeger's version

Notches - Strain-life approach

- The relationship between K_t, K_ε and K_σ are as follows:



Neuber's rule:
The geometric mean of the stress and strain concentration factors remain equal to k_t

⇨ Local stress lower than, and local strain greater than predicted with K_t above the yield stress

Nominally elastic behaviour in the notch

For nominally elastic behaviour

- Hooke's law

$$S = Ee$$

- The notch response then becomes:

$$\left\{\frac{(K_t S)^2}{E}\right\}_{applied\ load} = \{\sigma\varepsilon\}_{notch\ response}$$

- Components are usually designed such that nominal stress (S) and nominal strain (e) remain elastic:

➔ This form is most often used

For limited yielding in the notch

- Used when yielding occurs in the nominal stresses or strains
- Hooke's law can no longer be used

$$K_t^2 Se = \sigma\varepsilon$$

- Seeger's version
  - When very high stresses are present such that general yielding occurs
  - This version not used under normal circumstances

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Notches - Strain-life approach

- For lives > 1000 cycles, all three versions were close to actual lives
- For lives < 1000 ➔ Seeger's version more accurate
- Small notches have less effect than is indicated by K_t
  - A proposal is to use K_f instead of K_t in the Neuber equation

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Notches - Strain-life approach

- Procedure for applying the Neuber's rule in notch analysis
  - STEP 1. Initial loading
    - Define local stress strain curve. Use the **cyclic stress-strain** curve.

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{H'}\right)^{\frac{1}{n'}}$$

- Substitute in the Neuber equation:

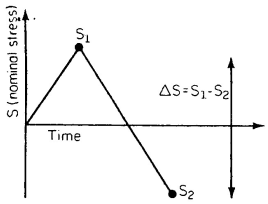
Notch response → $\sigma\varepsilon = \frac{(K_f S)^2}{E}$ ← Applied load in nominal stress

$$\sigma\left(\frac{\sigma}{E} + \left(\frac{\sigma}{H'}\right)^{\frac{1}{n'}}\right) = \frac{(K_f S)^2}{E}$$

- Solve for the local stress
- Graphically as shown on next slide

Notches - Strain-life approach

- STEP 2. Nominal Stress Reversal
  - Find change in nominal stress ($\Delta S = S_1 - S_2$)
  - **Hysteresis stress-strain curve**



$$\sigma\left(\frac{\sigma}{E}+\left(\frac{\sigma}{H'}\right)^{\frac{1}{n'}}\right)=\frac{(K_f S)^2}{E}$$

$$\varepsilon=\frac{\sigma}{E}+\left(\frac{\sigma}{H'}\right)^{\frac{1}{n'}}$$

$$\varepsilon_a=\frac{\sigma_f'}{E}(2N^*)^b+\varepsilon_f'(2N^*)^c$$

Calculate the mean stress–compensated endurance (fatigue life), N_f, as follows

$$N_{mi}^*=N_f\left(\frac{\sigma_a}{f(\sigma_a,\sigma_m)}\right)^{\frac{1}{b}}$$

Notches - Strain-life approach

- STEP 3. Successive cycles

The Neuber analysis result in a mean local stress. Apply mean stress correction. The Strain-life equation is used directly:

$$\varepsilon_a=\frac{\sigma_f'}{E}(2N^*)^b+\varepsilon_f'(2N^*)^c$$

The Neuber analysis may be implemented with a computer to evaluate the fatigue life of a notched component under variable loading

Example

RQC-100 steel is subjected to cycling with a strain amplitude of $\varepsilon_a = 0.005$ and a tensile mean stress of $\sigma_m = 150\ MPa$

How many cycles can be applied before fatigue cracking is expected?

Use Morrow, modified Morrow, SWT, and Walker mean stress correction methods

Material properties

Material properties of RQC-100 steel is found from Dowling Table 14.1

Parameter	Value	Unit
σ_o	683	MPa
σ_u	758	MPa
σ_{fB}	1 186	MPa
$\%RA$	64	
E	200 x 10^3	MPa
H'	903	MPa
n'	0.0905	
σ_f'	938	MPa
b	-0.0648	
ε_f'	1.38	
c	-0.704	

Morrow mean stress correction

- Demonstrate calculations in class
- Ramberg-Osgood

$$\varepsilon_a = \frac{\sigma_f'}{E}(2N^*)^b + \varepsilon_f'(2N^*)^c$$

Solved by Matlab to be $N^* = 5\ 635\ cycles$

- Mean-stress correction is then

$$N_f = N_{mi}^*\left(1 - \frac{\sigma_m}{\sigma_f'}\right)^{-\frac{1}{b}}$$

Variable amplitude loading: Stress

- Calculate nominal stress from loading
- Reorder stress history to begin and end with the peak or valley with the highest absolute value
- Calculate extrema
- Neuber equation for notch stress
- First stress: Use monotonic stress-strain curve
- To model memory effect of major cycle
  - Determine from-to stress points
  - Calculate stress and strain amplitudes & stress and strain
- Calculate zero-mean-stress-equivalent endurance
- Mean stress correction

Variable amplitude loading – Equations and material parameters

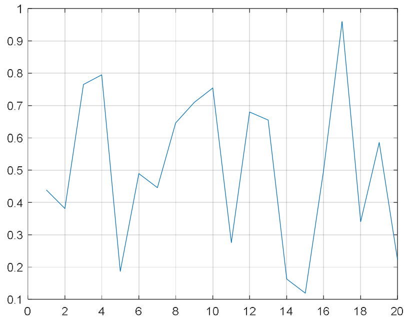
- Constitutive model: Stress-strain relationship. Use the Ramberg-Osgood equation. It needs: E, H' & n'. Get from Dowling Table 14.1 or from literature.

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{H'}\right)^{\frac{1}{n'}}$$

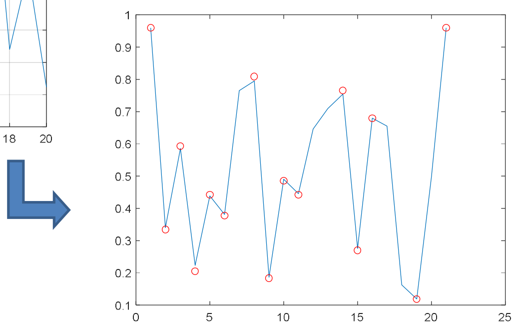
- Strain-life curve: It needs, $E, \sigma_f', b, \varepsilon_f'$ & c. Get from Dowling Table 14.1 or from literature. The pressure vessel code has long list of strain life parameters.

$$\varepsilon_a = \frac{\sigma_f'}{E}(2N^*)^b + \varepsilon_f'(2N^*)^c$$

Reorder stress history



Stress history reordered to start and end at the peak or valley with the highest absolute value
Cycle counting done on original history, or history of extrema



Monotonic stress-strain response

Ramberg-Osgood monotonic

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{H'}\right)^{\frac{1}{n'}} = f(\sigma)$$

Ramberg-Osgood-cyclic

$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}} = f(\sigma_a)$$

Neuber's rule

$$\varepsilon\sigma = \frac{(k_t S)^2}{E}$$

$$\varepsilon = \frac{(k_t S)^2}{\sigma E} = g(S, \sigma)$$

Neuber's rule

$$\varepsilon_a \sigma_a = \frac{(k_t S_a)^2}{E}$$

$$\varepsilon_a = \frac{(k_t S_a)^2}{\sigma_a E} = g(S_a, \sigma_a)$$

Endpoint stress and strain

- Endpoint of the stress and strain is then:

$$\sigma_{i+1} = \sigma_i + \psi\Delta\sigma$$

$$\varepsilon_{i+1} = \varepsilon_i + \psi\Delta\varepsilon$$

- Calculate the means:

$$\sigma_{mi} = \frac{(\sigma_i + \sigma_{i+1})}{2}$$

$$\varepsilon_{mi} = \frac{(\varepsilon_i + \varepsilon_{i+1})}{2}$$

Strain-life equation & Palmgren-Miner's rule

The zero-mean-stress-equivalent fatigue life is:

$$\varepsilon_a = \frac{\sigma_f'}{E}(2N^*)^b + \varepsilon_f'(2N^*)^c$$
$$\frac{(k_t S_a)^2}{\sigma_a E} = \frac{\sigma_f'}{E}(2N^*)^b + \varepsilon_f'(2N^*)^c$$
$$f_{errsl} = \frac{\sigma_f'}{E}(2N^*)^b + \varepsilon_f'(2N^*)^c - \frac{(k_t S_a)^2}{\sigma_a E}$$

The mean-stress compensated fatigue life is:

$$N_f = N_{mi}^*\left(1 - \frac{\sigma_m}{\sigma_f'}\right)^{-\frac{1}{b}}$$

Damage:

$$D = \sum \frac{n_i}{N_{f,i}}$$

Number of repetitions

$$B_f = \frac{1}{D}$$

Discuss problem in handout notes

- Discuss Section 14.9 of the handout notes in class
  - Purpose, to demonstrate application of the technique with Matlab code