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S-N curves and fatigue damage

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Introduce S-N curves to the student and how it is used

Agenda:

- Summary of values from constant amplitude testing
- S-N curve for steels
- S_r-N curve for welds

Fatigue

Fatigue is the following process

- Repeated loads cause cyclic stresses
- Lead to microscopic physical damage
- Microscopic damage can accumulate at stresses well below S_{ut}
- Microscopic damage develops into crack or other macroscopic damage that leads to failure

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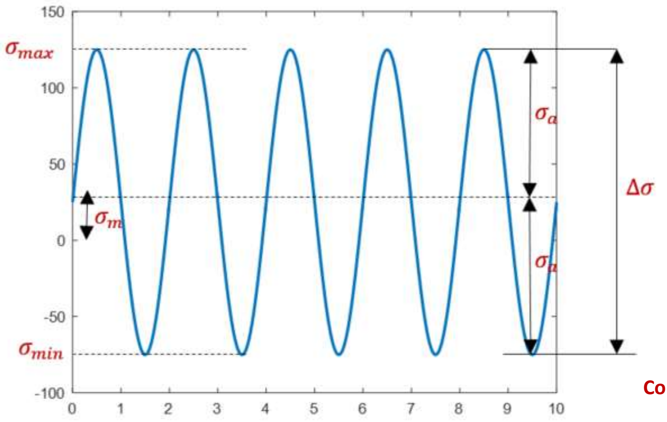
Constant amplitude alternating stress loading parameters



Note, for nominal stress away from notches, the symbol S is used

Maximum stress: σ_{max}

Minimum stress: σ_{min}

Stress range:

$$\Delta\sigma = \sigma_{max} - \sigma_{min}$$

Stress amplitude:

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

Mean stress:

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

Stress ratio:

$$R = \frac{\sigma_{min}}{\sigma_{max}}$$

Amplitude ratio:

$$A = \frac{\sigma_a}{\sigma_m}$$

Completely reversed:
$R = -1; A = \infty$
Zero to max: $R = 0; A = 1$
Zero to min: $R = \infty;\ A = -1$

Relationships for the constant amplitude signal

- Stress amplitude vs maximum stress

$$\begin{aligned}\sigma_a &= \frac{\Delta\sigma}{2}\\ &= \frac{\sigma_{max}}{2}(1-R)\end{aligned}$$

- Mean stress vs maximum stress

$$\begin{aligned}\sigma_m &= \frac{\sigma_{max}+\sigma_{min}}{2}\\ &= \frac{\sigma_{max}+R\sigma_{max}}{2}\\ &= \frac{\sigma_{max}}{2}(1+R)\end{aligned}$$

- Stress ratio vs amplitude ratio

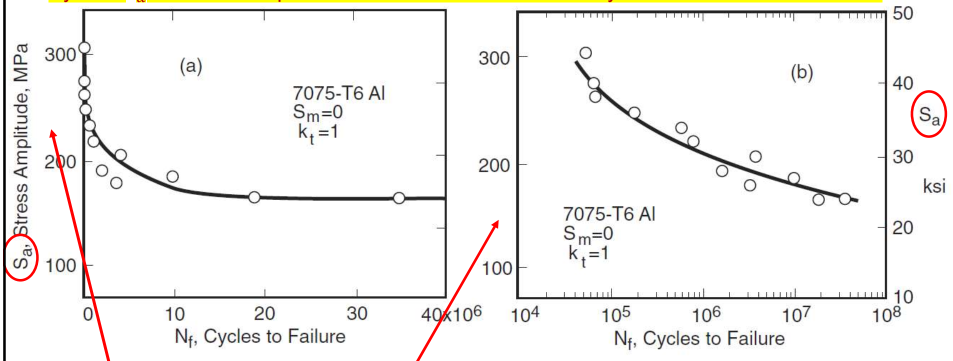
$$\begin{aligned}R &= \frac{\sigma_{min}}{\sigma_{max}}\\ &= \frac{\sigma_m-\sigma_a}{\sigma_m+\sigma_a}\\ &= \frac{1-\frac{\sigma_a}{\sigma_m}}{1+\frac{\sigma_a}{\sigma_m}}\\ &= \frac{1-A}{1+A}\end{aligned}$$

- Amplitude ratio vs stress ratio

$$A = \frac{1-R}{1+R}$$

Stress vs Life: 7075-T6 Al – rotating bending test

Symbol S_a for stress amplitude indicate nominal stress away from a stress concentration



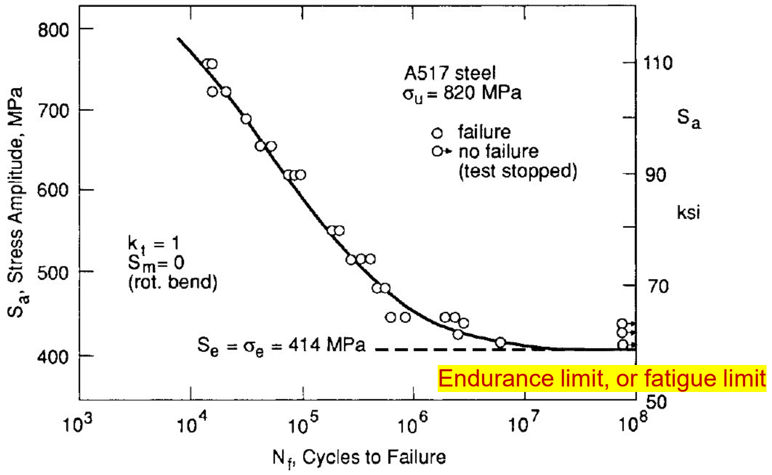
Linear scale

Logarithmic scale

Stress amplitude, maximum stress or stress range can be used for the ordinate, just modify with mean stress effects

Note, there is no distinct endurance limit, also called fatigue limit

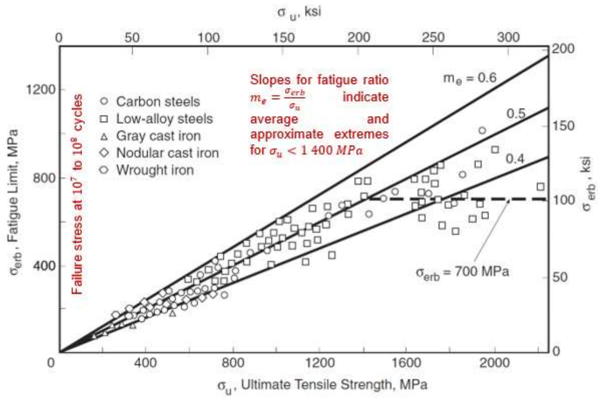
Stress vs fatigue life: A517 steel – rotating bending test



Endurance limit, or fatigue limit

Rotating bending fatigue limit σ_{erb} vs ultimate tensile strength σ_u

σ_{erb} is the fatigue limit from a rotating bending test

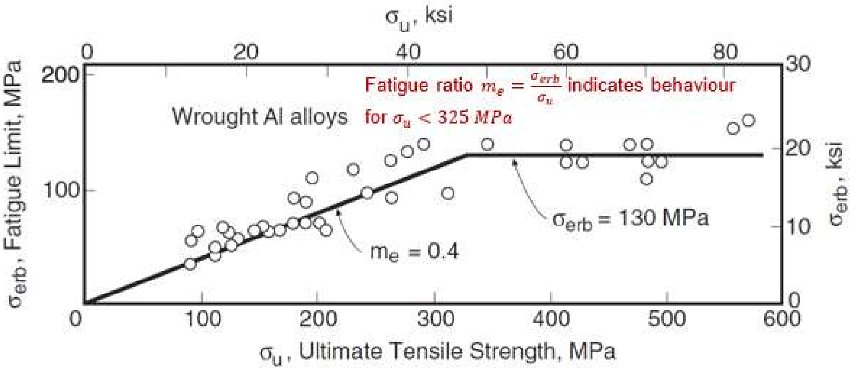


Slopes $m_e = \frac{\sigma_{erb}}{\sigma_u}$ indicated average and approximate extremes for $\sigma_u < 1\,400\ MPa$

Source: (Dowling, 2013, pp. 442, Figure 9.24)

Figure 6: Rotating bending fatigue limits, or failure stresses from polished specimens

Rotating bending test of wrought aluminium alloys



Note: Fatigue strength at 5×10^8 cycles for alloys including 1100, 2014, 2024, 3003, 5052, 6061, 6063, and 7075.

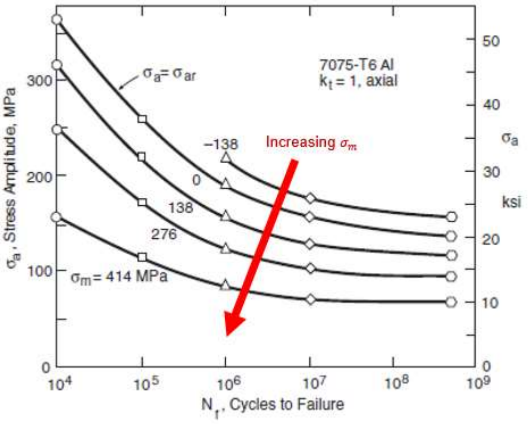
Source: (Dowling, 2013, pp. 443, note permission applicable)

Glass-fibre reinforced thermoplastics

- Typically tested under zero-to-maximum tension or bending
- In this case the maximum stress is used

$$\sigma_{max} = \sigma_u\left(1 - 0.1\log_{10} N_f\right)$$

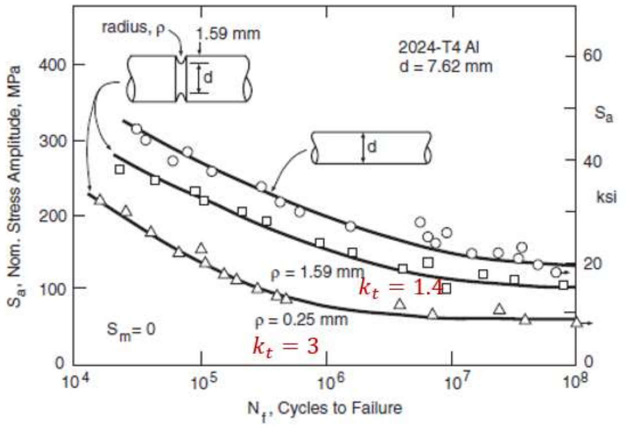
Mean stress effect on aluminium alloy



(Dowling, 2013, p. 444)

What does notches do

Note that the nominal stress amplitude is shown in the ordinate. If the stress in the notch was calculated and shown, the fatigue curves will be closely spaced.



Source: (Dowling, 2013, p. 444)

Environmental effects & frequency of cycling

- Selfstudy: Dowling Chapter 9.6.2
  - Chemical environments, temperature and the time effect thereof that result in reduced strength under low frequency testing, not because of the strain rate dependence, but, because of delayed affects at the crack tip and elsewhere
  - Temperature effects and hysteresis in polymers under cyclic loading

Effects of microstructure on fatigue behaviour

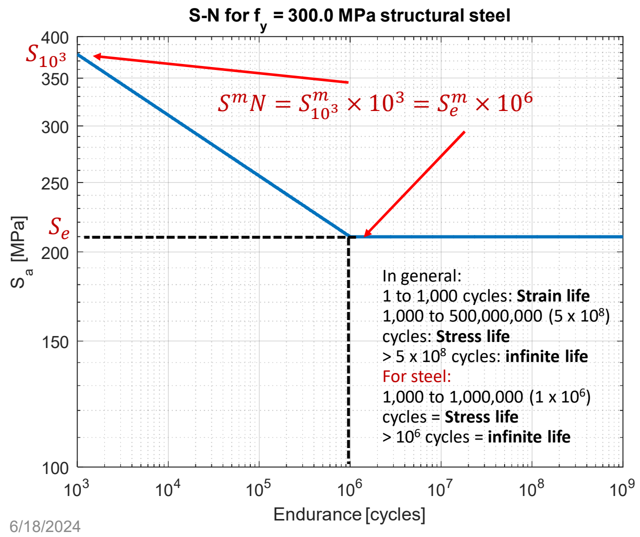
- Selfstudy: Read Dowling Chapter 9.6.3. and take note of the following:
  - Reducing size of inclusions and voids – enhance fatigue resistance
  - Reducing grain size – enhance fatigue resistance
    - Thorough annealing result in larger grains size that lowers fatigue resistance
  - Presence of a dense network of dislocations – enhance fatigue resistance
    - Effect of cold work by drawing
  - Lower fatigue resistance normal to the long direction after rolling
  - In laminated structures – higher fatigue resistance where larger numbers of fibres are parallel to the applied stress, and lower for stresses normal to the plane of laminated structures

Effect of residual stress on fatigue behaviour

Selfstudy: Read Dowling Chapter 9.6.4. and take note of the following:

- Compressive residual stresses are beneficial
- Shot peening: bombarding the surface with small steel or glass shot, producing a compressive surface layer
- Presetting: Bending to cause elongation at an outer point on a beam causing a compressive residual stress on release
  - Note the opposite effect on the side subject to compressive stress
- Machining: Smoother surfaces improve fatigue resistance
  - Some machining procedures can be harmful due to tensile residual stresses
- Surface treatments:
  - May alter the microstructure, chemical composition, and/or residual stress of the surface
  - Plating (nickel, chromium)
    - Introduce tensile residual stress – lowers fatigue strength
    - Deposited material may have poorer resistance to fatigue than base metal
    - Shot peen after plating if required
- Welding:
  - Introduce notches and other stress raisers
  - Have tensile residual stress
  - Unusual microstructure distributions

S-N curve for steel – from rotating bending tests



$$S^m N = S_{10^3}^m \times 10^3 = S_e^m \times 10^6$$

In general:
1 to 1,000 cycles: **Strain life**
1,000 to 500,000,000 (5×10^8) cycles: **Stress life**
> 5×10^8 cycles: **infinite life**
For steel:
1,000 to 1,000,000 (1×10^6) cycles = **Stress life**
> 10^6 cycles = **infinite life**

For steel:

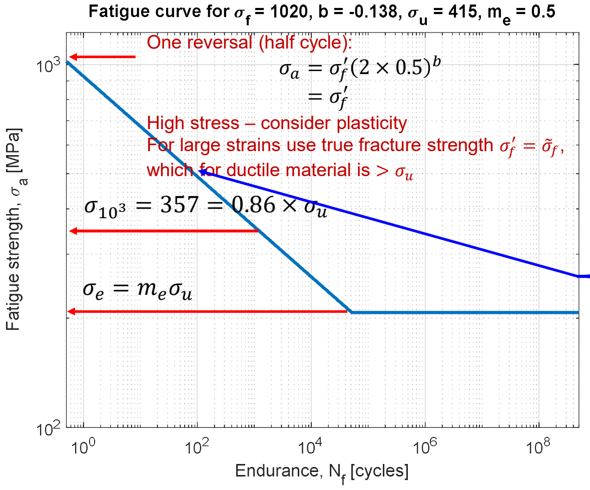
$$S_{10^3} = 0.9 f_{ut}$$
$$S_e = 0.5 f_{ut}$$

This is for a 50% probability of survival

To determine m:

$$S_{10^3}^m \times 10^3 = S_e^m \times 10^6$$
$$\left(\frac{S_{10^3}}{S_e}\right)^m = 10^3$$
$$m = \frac{3}{\log_{10}\frac{S_{10^3}}{S_e}}$$

$\sigma_a = \sigma_f'\left(2N_f\right)^b$



One reversal (half cycle):

$$\sigma_a = \sigma_f'(2 \times 0.5)^b$$
$$= \sigma_f'$$

High stress – consider plasticity
For large strains use true fracture strength $\sigma_f' = \tilde{\sigma}_f$, which for ductile material is $> \sigma_u$

$$\sigma_{10^3} = 357 = 0.86 \times \sigma_u$$

$$\sigma_e = m_e \sigma_u$$

S-N data approximate straight line:

$$\sigma_a = C + D \log N_f$$
$$= A N_f^b$$

Following then applies at any point on the S-N curve:

$$\sigma_1 = A N_1^b$$
$$\sigma_2 = A N_2^b$$
$$\sigma_i = A N_i^b$$
$$\frac{\sigma_1}{\sigma_2} = \left(\frac{N_1}{N_2}\right)^b$$
$$N_1 = \left(\frac{\sigma_2}{\sigma_1}\right)^{\frac{1}{b}} N_2$$

Can be rewritten:

$$\sigma_a = \sigma_f'\left(2N_f\right)^b$$
$$A = 2^b \sigma_f'$$

Short fatigue life

- High stresses are involved
- Plastic strains could also be present
- In this case:
  - Amplitudes of true stress are needed for quite large strains
  - Then use, $\sigma_f' = \tilde{\sigma}_f$, the true fracture strength from tension test
  - The true fracture strength for ductile materials is more than the ultimate tensile strength

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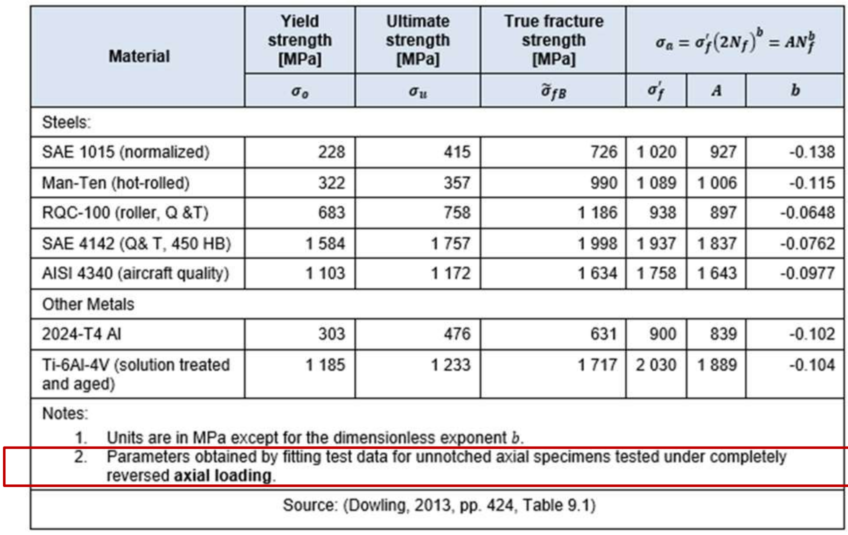
Constants for stress-life curves for ductile engineering metals on unnotched axial specimens



Material	Yield strength [MPa]	Ultimate strength [MPa]	True fracture strength [MPa]	$\sigma_a = \sigma_f'(2N_f)^b = AN_f^b$		
	σ_o	σ_u	$\tilde{\sigma}_{fB}$	σ_f'	A	b
Steels:						
SAE 1015 (normalized)	228	415	726	1 020	927	-0.138
Man-Ten (hot-rolled)	322	357	990	1 089	1 006	-0.115
RQC-100 (roller, Q &T)	683	758	1 186	938	897	-0.0648
SAE 4142 (Q& T, 450 HB)	1 584	1 757	1 998	1 937	1 837	-0.0762
AISI 4340 (aircraft quality)	1 103	1 172	1 634	1 758	1 643	-0.0977
Other Metals						
2024-T4 Al	303	476	631	900	839	-0.102
Ti-6Al-4V (solution treated and aged)	1 185	1 233	1 717	2 030	1 889	-0.104

Notes:

1. Units are in MPa except for the dimensionless exponent b.
2. Parameters obtained by fitting test data for unnotched axial specimens tested under completely reversed **axial loading**.

Source: (Dowling, 2013, pp. 424, Table 9.1)

Using the idea

Construct the S-N curve for a material with completely reversed stress amplitude $S_1 = 500\ MPa$ at $N_1 = 10^3$ and endurance limit $S_e = 150\ MPa$ at $N_e = 10^6$ cycles. What is the endurance at 300 MPa?

Problem is done in the notes handed out.

Solution

- Two points on the S-N curve are given:

$$S_1^m N_1 = S_2^m N_2$$

- The exponent m can be determined as:

$$\left(\frac{S_1}{S_2}\right)^m = \frac{N_2}{N_1}$$

$$m \log \frac{S_1}{S_2} = \log \frac{N_2}{N_1}$$

$$m = \frac{\log \frac{N_2}{N_1}}{\log \frac{S_1}{S_2}}$$

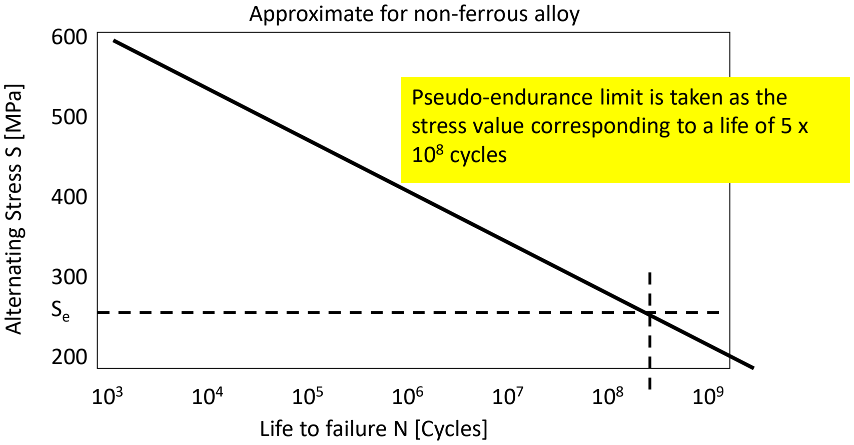
$$= 5.74$$

$$N_2 = \left(\frac{S_1}{S_2}\right)^m N_2$$

$$= \left(\frac{500}{300}\right)^{5.74} 1000$$

$$= 18\,767\ cycles$$

Non-ferrous alloys: Pseudo-endurance limit at 5 x 10^8



Pseudo-endurance limit is taken as the stress value corresponding to a life of 5 x 10^8 cycles

Fatigue ratio

Ratio of endurance limit to ultimate tensile strength

For steel f_r (or m_e) varies between 0.35 to 0.6

Most steels with S_{ut} below 200 ksi (≈1,400MPa) have a fatigue ratio of 0.5

$$f_r = m_e = \frac{S_e}{\sigma_u}$$

Endurance limit

As function of **surface hardness** of material:

$$S_e = \begin{cases} 0.25BHN\ ksi & for\ BHN \leq 400 \\ 100\ ksi & for\ BHN > 400 \end{cases}$$

In terms of ultimate tensile strength:

Steel

$$S_e = \begin{cases} 0.5S_{ut} & for\ S_{ut} \leq 200\ ksi\ (1\,400\ MPa) \\ 100\ ksi\ (700\ MPa) & for\ S_{ut} > 200\ ksi\ (1\,400\ MPa) \end{cases}$$

Cast Iron + Cast Steels:

$$S_e = \begin{cases} 0.45S_{ut} & for\ S_{ut} \leq 600\ MPa \\ 275\ MPa & for\ S_{ut} > 600\ MPa \end{cases}$$

$$m_e = \frac{S_e}{\sigma_u}$$

Material	m_e	N_f **[cycles]**
Aluminium alloys	0.40	5×10^8
Low- and intermediate-strength steels	0.50	10^6
Cast irons	0.40	10^7
Wrought magnesium alloys	0.35	10^8
Titanium alloys	0.5	10^7

Source: (Dowling, 2013, p. 502)

Pure shear endurance limit

- Pure shear endurance limit:

$$\tau_{er} = \frac{\sigma_{erb}}{\sqrt{3}}$$
$$= 0.577\sigma_{erb}$$

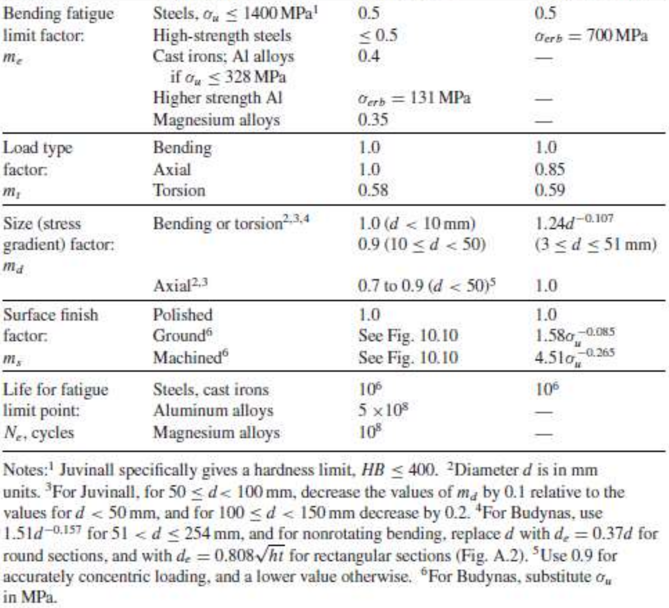


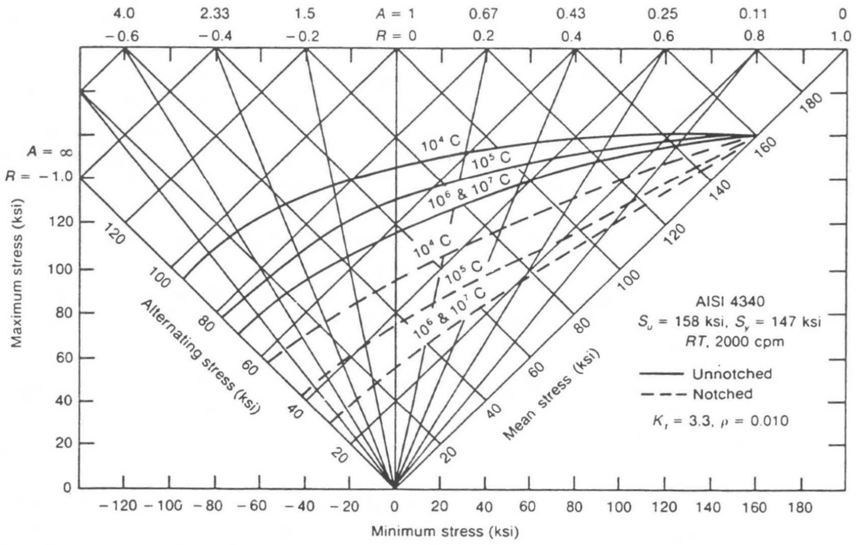
Table 10.1 Parameters for Estimating Fatigue Limits

Parameter	Applicability	Juvinall (2006)	Budynas (2011)
Bending fatigue limit factor: m_e	Steels, $\sigma_u \leq 1400$ MPa[1]	0.5	0.5
	High-strength steels	≤ 0.5	$\sigma_{erb} = 700$ MPa
	Cast irons; Al alloys if $\sigma_u \leq 328$ MPa	0.4	—
	Higher strength Al	$\sigma_{erb} = 131$ MPa	—
	Magnesium alloys	0.35	—
Load type factor: m_t	Bending	1.0	1.0
	Axial	1.0	0.85
	Torsion	0.58	0.59
Size (stress gradient) factor: m_d	Bending or torsion[2,3,4]	1.0 ($d < 10$ mm) 0.9 ($10 \leq d < 50$)	$1.24d^{-0.107}$ ($3 \leq d \leq 51$ mm)
	Axial[2,3]	0.7 to 0.9 ($d < 50$)[5]	1.0
Surface finish factor: m_s	Polished	1.0	1.0
	Ground[6]	See Fig. 10.10	$1.58\sigma_u^{-0.085}$
	Machined[6]	See Fig. 10.10	$4.51\sigma_u^{-0.265}$
Life for fatigue limit point: N_e, cycles	Steels, cast irons	10^6	10^6
	Aluminum alloys	5×10^8	—
	Magnesium alloys	10^8	—

Notes:[1] Juvinall specifically gives a hardness limit, $HB \leq 400$. [2]Diameter d is in mm units. [3]For Juvinall, for $50 \leq d < 100$ mm, decrease the values of m_d by 0.1 relative to the values for $d < 50$ mm, and for $100 \leq d < 150$ mm decrease by 0.2. [4]For Budynas, use $1.51d^{-0.157}$ for $51 < d \leq 254$ mm, and for nonrotating bending, replace d with $d_e = 0.37d$ for round sections, and with $d_e = 0.808\sqrt{ht}$ for rectangular sections (Fig. A.2). [5]Use 0.9 for accurately concentric loading, and a lower value otherwise. [6]For Budynas, substitute σ_u in MPa.

Derivation of constants for other materials

- Not so simple because of potential non-linear S-N characteristics
- Linear approximation used in most applications due to empirical data used in analysis – statistical errors exists already



Haigh diagram (Master diagram) for AISI 4340 steel (Bannantine *et all*, 1990:7)

BS 7608, AWS D1.1M & EN 1993-1-9

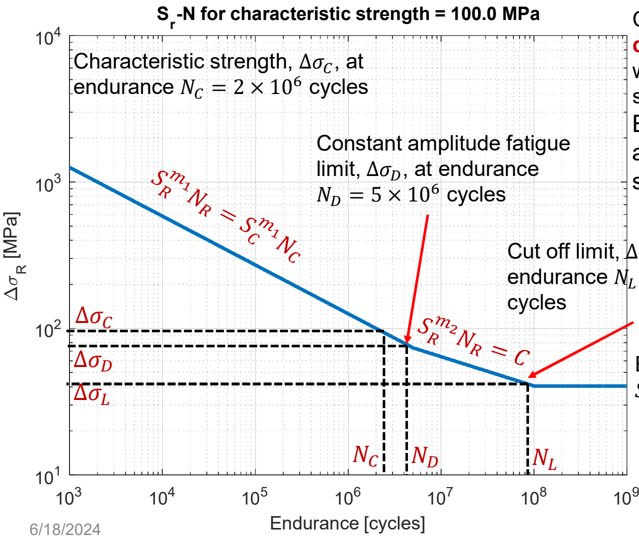
These standards use an S-N curve with stress range vs. endurance

$S_R - N$ curve

This is because weld material is not mean stress sensitive

More about this later

Typical EN 1993-1-9 $S_r - N$ fatigue curve



Curve is characterized by the **characteristic strength, $\Delta\sigma_C$,** which is provided by the standards

Every fatigue curve is for a specified probability of survival

Exponents, or slopes of the $S_r - N$ curve are:

$m_1 = 3$
$m_2 = 5$

Test the ideas

Calculate the constant amplitude fatigue limit and the cut-off limit for a detail category 100, i.e., the characteristic strength is $\Delta\sigma_C = 100$ MPa

Solution

The following equation applies for the section to the constant amplitude fatigue limit:

$$\Delta\sigma_C^{m_1} N_C = \Delta\sigma_D^{m_1} N_D$$

$$\Delta\sigma_D = \left(\Delta\sigma_C^{m_1} \frac{N_C}{N_D}\right)^{\frac{1}{m_1}}$$

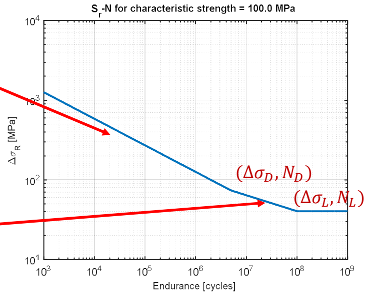
$$= \Delta\sigma_C \cdot \left(\frac{N_C}{N_D}\right)^{\frac{1}{m_1}}$$

$$= 0.74\Delta\sigma_C$$

$$\Delta\sigma_L = \Delta\sigma_D \left(\frac{N_D}{N_L}\right)^{\frac{1}{m_2}}$$

$$= 0.55\Delta\sigma_D$$

$$= 0.41\Delta\sigma_C$$



Damage

- Most service loading histories have variable amplitude
- Sometimes stochastic of nature (random probability distribution, may be analysed statistically but cannot be predicted precisely)
- The following aspects need to be addressed:
  - Nature of fatigue damage and how it can be related to load history
  - Damage summation methods
  - Cycle counting techniques to recognise damaging events
  - Crack propagation behaviour under variable amplitude loading
  - How to deal with service load histories

Variable amplitude loading

Fatigue is that tendency of a material to fail due to a crack that initiates and propagates

Definition of fatigue damage

- Is the measurable propagation portion of cracks
  - Damage is directly related to crack length ⇨ it is observable, measurable
  - Inspection intervals used to monitor crack growth
- Initiation phase
  - Mechanisms on microscopic level (dislocations, slip bands, micro-cracks, etc.)
  - Only measurable in highly controlled laboratory environment

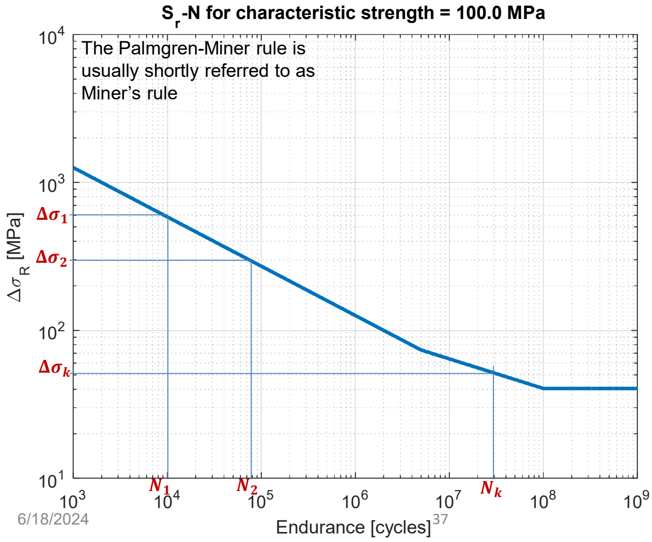
⇨ **Most damage summing methods during initiation phase empirical of nature**

Stress spectrum

Note, we use S for nominal and σ for local stress. Equations apply for both.

Stress amplitude σ_a[MPa]	**Stress mean σ_m[MPa]**	**Number of cycles n**	**Stress range $\Delta\sigma = 2\sigma_a$**	**Goodman correction $\sigma_{ar} = \frac{\sigma_a}{1-\frac{\sigma_m}{\sigma_{ut}}}$**	**Endurance, for given $P_{survival}$ N**	**Damage $d_i = \frac{n_i}{N_i}$**
100	0	1 000				
200	-100	5 000				
50	50	100 000				
300	100	50 000				
Total damage, $D = \sum d_i$						
Number of repetitions, $B_f = \frac{1}{D}$						
Fatigue life, $L = Period \times B_f$						

Linear damage rule – Palmgren-Miner rule



The Palmgren-Miner rule is usually shortly referred to as Miner's rule

Damage per stress range:

$$D_i = \frac{n_i}{N_i}$$

Damage for k different stress ranges:

$$D_T = \sum \frac{n_i}{N_i}$$

Failure occurs when:

$$D_T = 1.0$$

Values can vary between 0.5 to 2 in practice

Convenient to calculate repetitions, B_f, that a stress spectrum can be applied:

$$B_f \left[\sum \frac{n_i}{N_i} \right] = 1$$

$$\therefore B_f = \frac{1}{\sum \frac{n_i}{N_i}}$$

Shortcomings of Palmgren-Miner rule

- It does not consider sequence effects for plastic deformation & residual stress!
- It is amplitude independent
  - Predicts that the rate of damage accumulation is independent of stress level
  - Observed behaviour show that:
    - At high strain amplitudes, cracks will initiate in a few cycles
    - At low strain amplitudes, almost all the life is spent initiating a crack

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Non-linear damage theories

- Practical problems
  - Require material and shaping constants which must be determined experimentally
  - Sequence effects must be tested for
- Cannot be guaranteed that these methods will be more accurate than Palmgren-Miner's rule

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Summary – Cumulative damage techniques

- Use the Palmgren-Miner rule
- Non-linear techniques not significantly more accurate
- Damage summation techniques must account for load sequence effects (mean stress, residual stress) ⇨ use strain life to account for initially high stresses

Safety factors for S-N curves

- Safety factor in stress

$$X_S = \frac{\sigma_{a1}}{\sigma_{actual}}$$

- Safety factor in life

$$X_N = \frac{N_{f2}}{N_{actual}}$$

Conclusion

- Fatigue curve is a log-log presentation of endurance for a given stress amplitude or range
- Constructed from known points on the curve
- Is for specific significance level for probability of survival
  - Fatigue is probabilistic of nature
- Damage is calculated from the number of cycles applied and the endurances on the S-N curve
- More in the fatigue curves later in the course