Multiaxial Fatigue

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Cracking Observations

- Multiaxial fatigue theories must be consistent with physical observations - purpose of fatigue analysis is to reconcile analytical approaches with physical observations
- Fatigue crack nucleation
  - Result of cyclic loading causing to-and-fro slip
  - Slips results in the development of weakened bands - slip bands
  - Cracks initiate at the slip bands
- Slip bands
  - First occur in those grains whose slip planes are most closely aligned with the plane of maximum shear strain

- Fatigue crack growth occur in two stages
  - Stage 1: Growth includes nucleation and early growth spent on shear planes
    - Both shear and normal stresses and strains on the crack face are important
    - Normal stresses and strains tend to reduce amount of Stage 1 growth
    - Initiate here even in out-of-phase loading
  - Stage 2: Crack growth occur on planes that are oriented perpendicular to the maximum principal stress range (Mode I growth)
  - Portion of life spent on Stage 1 and Stage 2 planes shown to depend on material type, loading mode, strain amplitude

Multiaxial Fatigue Theories

- Octahedral effective Stress/Strain Approach
- Sines' Model
  - Maximum range of shear stress criterion
  - Equivalent strain range criterion
- Maximum range of shear stress criterion
- Maximum range of shear strain criterion
- Critical plane approach

Multiaxial Fatigue

- Engineering elements subjected to complicated states of stress and strain
- Complex stress states is where the three principal stresses are non-proportional or whose directions change during a loading cycles
  - Geometric discontinuities
  - Combined bending and torsion
- Theories are continuously developed and modified to account for multiaxial fatigue

Stress State

- Multiaxial fatigue requires understanding of stress and strain state
  - Strain gauges
  - Finite element analysis
- Multiaxial fatigue
  - Directions of the principal stresses at critical location change during loading cycles = f(time)
  - Loading situations arise where the three principal stresses are non-proportional

Octahedral stress approach

- When cyclic loads are:
  - Completely reversed and have the same frequency
  - Either in-phase (0°) or 180° out-of-phase with one another
- Effective stress amplitude:

$$\bar{\sigma}_a = \frac{1}{\sqrt{2}}\sqrt{(\sigma_{1a}-\sigma_{2a})^2+(\sigma_{2a}-\sigma_{3a})^2+(\sigma_{3a}-\sigma_{1a})^2}$$
$$= \frac{1}{\sqrt{2}}\sqrt{\left(\sigma_{xa}-\sigma_{ya}\right)^2+\left(\sigma_{ya}-\sigma_{za}\right)^2+(\sigma_{za}-\sigma_{xa})^2+6(\tau_{xya}^2+\tau_{yza}^2+\tau_{zxa}^2)}$$

- Effective mean stress:

$$\bar{\sigma}_m = \sigma_{1m}+\sigma_{2m}+\sigma_{3m}$$

Mean-stress correction

- Mean stress correction:
  - Morrow:

$$\frac{\sigma_a}{\sigma_{ar}}+\frac{\sigma_m}{\sigma_f'}=1$$
$$\sigma_{ar}=\frac{\sigma_a}{1-\frac{\sigma_m}{\sigma_f'}}$$
$$\sigma_{ar}=\frac{\bar{\sigma}_a}{1-\frac{\bar{\sigma}_m}{\sigma_f'}}$$

  - Do similarly for other mean stress rules

- Pure shear $\tau_{xya} = \tau_{xym} \neq 0$,
  - Octahedral effective stress:

$$\bar{\sigma}_a = \frac{1}{\sqrt{2}}\sqrt{6\left(\tau_{xya}^2\right)}$$

$$= \sqrt{3}\tau_{xya} \quad \bar{\sigma}_m = 0$$

  - Mean stress taken as 0, even if there is a mean stress present – this agrees with experimental observation

Effective strain range criterion

- Used in the Low Cycle Fatigue regime
- Requires calculation of an equivalent strain range

$$\Delta\bar{\varepsilon} = \frac{\sqrt{2}}{3}\left((\Delta\varepsilon_{11} - \Delta\varepsilon_{22})^2 + (\Delta\varepsilon_{22} - \Delta\varepsilon_{33})^2 + (\Delta\varepsilon_{11} - \Delta\varepsilon_{33})^2 + 6[\Delta\varepsilon_{12}^2 + \Delta\varepsilon_{23}^2 + \Delta\varepsilon_{31}^2]\right)^{0.5} \leq A$$

- Terms in brackets maximized with respect to time where:

$$\textit{Strain differences} \quad \Delta\varepsilon_{ij} = \varepsilon_{ij}(t_1) - \varepsilon_{ij}(t_2)$$

$$\textit{Strain component at } t_1 \quad \varepsilon_{ij}(t_1)$$

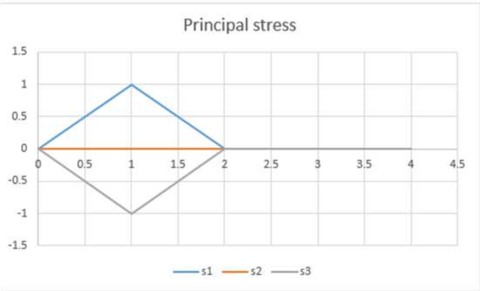
$$\textit{Strain component at } t_2 \quad \varepsilon_{ij}(t_2)$$

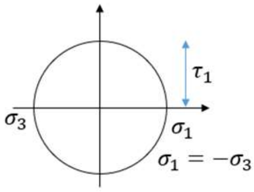
Equivalent Strain range criterion

- For proportional loading
  - Equal the octahedral shear strain, though, in general not equal to the octahedral shear strain theory
- Limitations
  - Does not predict a dependence of the fatigue life on hydrostatic stress
  - Only dependent on the strain at two instants in time ⇒ Path independence which was shown experimentally as incorrect
    - See the two following figures

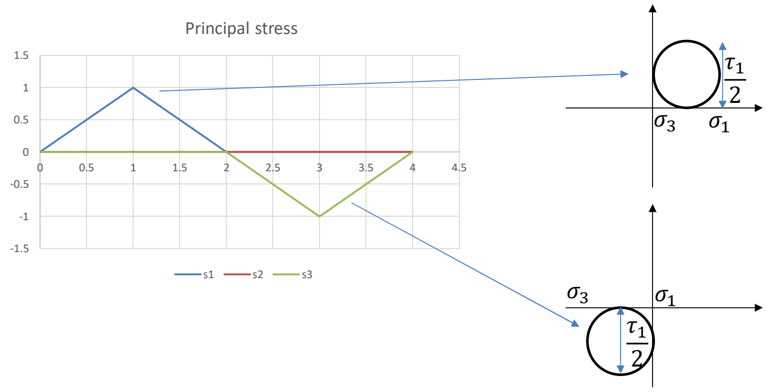
Shortcoming of shear stress and strain range criteria: Case 1

t	σ_1	σ_2	σ_3
0	0	0	0
1	1	0	-1
2	0	0	0
3	0	0	0
4	0	0	0





Shortcoming of shear stress and strain range criteria: Case 2



Shortcoming: discussion

- Different fatigue life results because in Case 1 the single cycle had a large shear stress, and, in Case 2, two cycles of smaller amplitude were experienced
- The Maximum shear stress range criterion predicts the same life for these two signals because the ranges are the same

Sine's Model

- Sines observed the following
  - Torsional mean stress did not affect fatigue life of a specimen subjected to alternating torsion or bending stresses
  - Tensile mean stress reduces the fatigue life of a component subjected to cyclic torsional loading
  - Tensile mean stress decrease and compressive mean stress increase fatigue life of specimen under uniaxial loading

Sine's Model

- Sine's relationship

$$\frac{1}{3}\left((P_1 - P_2)^2 + (P_2 - P_3)^2 + (P_1 - P_3)^2\right)^{0.5} + \alpha\left(S_x + S_y + S_z\right) \leq A$$

Where:

P_1, P_2, P_3	Amplitudes of the alternating principal stresses
S_x, S_y, S_z	Orthogonal (any coordinate system) static (mean) stresses
α	Material constant which gives variation of the permissible range of stress with static stress
A	Material constant proportional to the reversed fatigue strength

Sine's Model

- The constants can be easily found
  - From a completely reversed uniaxial test
    - Find A
  - For 0 to F loading
    - Find α
- Disadvantages of Sines' model
  - Only dependent on the strain at two instants in time $\Rightarrow$ Path independence which was shown experimentally as incorrect
  - Limited to applications in which the principal axes of the alternating stress components are fixed to the body
    - To overcome this - Modification: Maximum range of shear stress criterion

Maximum range of shear stress criterion

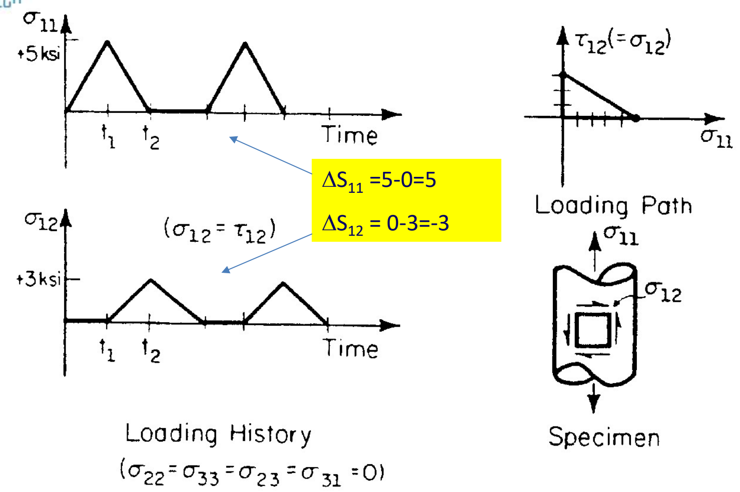
- This criterion eliminates the restriction that the principal axes of the body must remain fixed
- The first term in the Sine's model is modified:

$$\frac{1}{6}\left((\Delta S_{11}-\Delta S_{22})^2+(\Delta S_{22}-\Delta S_{33})^2+(\Delta S_{11}-\Delta S_{33})^2+6[\Delta S_{12}^2+\Delta S_{23}^2+\Delta S_{31}^2]\right)^{0.5}+\alpha\left(S_x+S_y+S_z\right)\leq A$$

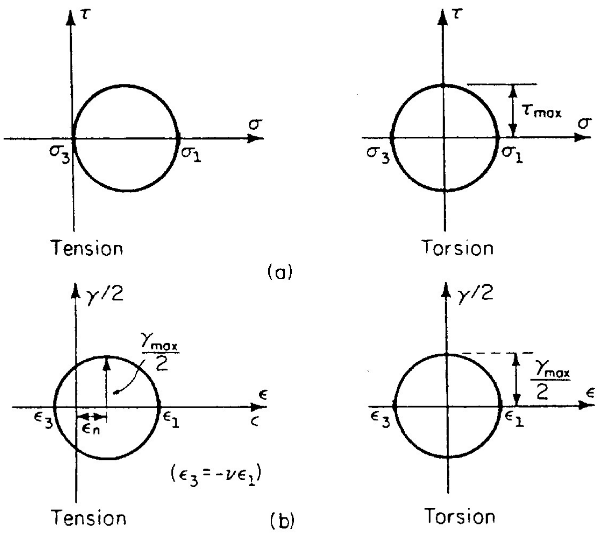
- Where the term in brackets is maximized with respect to time and where:

$\Delta S_{ij}=\sigma_{ij}(t_1)-\sigma_{ij}(t_2)$	Stress differences
$\sigma_{ij}(t_1)$	Stress component at time t_1
$\sigma_{ij}(t_2)$	Stress component at time t_2

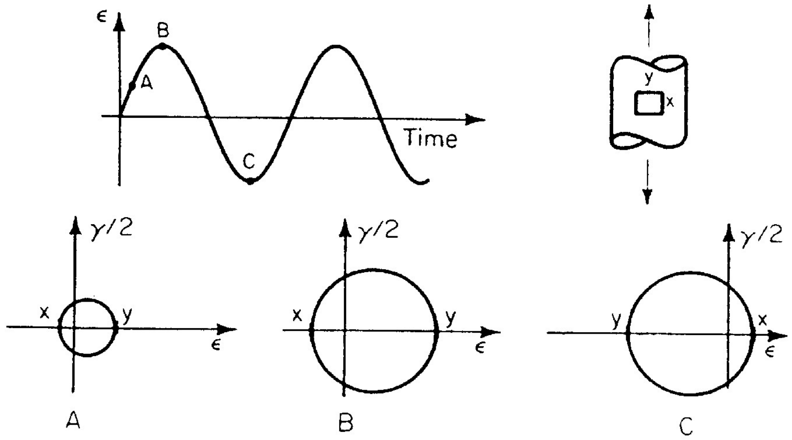
Maximum range of shear stress criterion-Example



Mohr's circle representation of tension and torsion

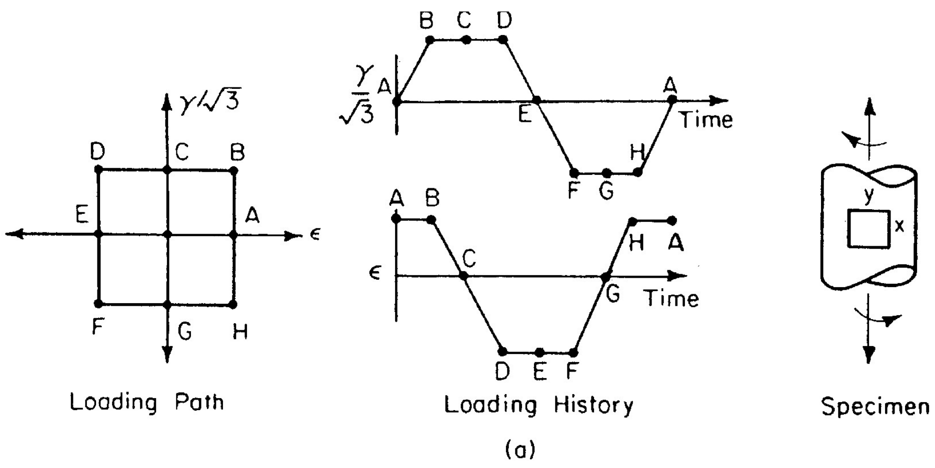


Mohr's circle presentation of uniaxial loading

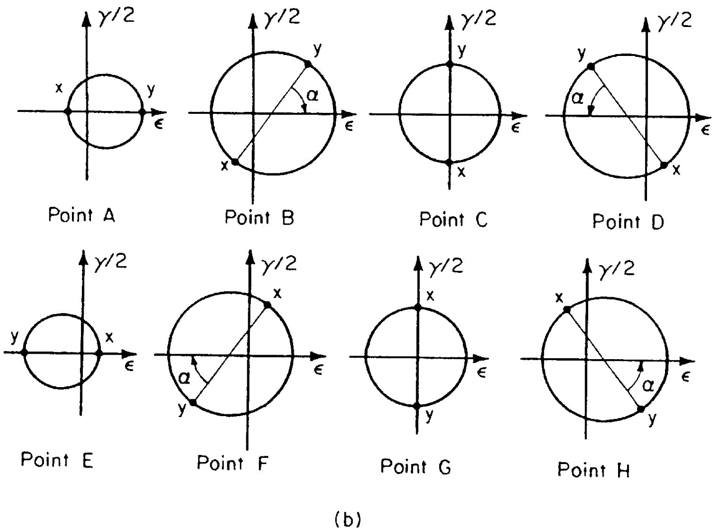


Size changes. Direction of principal stress and strain remains constant for uniaxial loading.

Mohr's circle presentation of combined tension and torsion



Mohr's circels - Combined tension and torsion



Conclusions from combined tension and torsion loading

- Direction of principal axis changes - Points X and Y not at the same points all the time.
- Ratio of shear strain to normal strain changes - nonproportional loading.
- Combined out-of-phase loading is more damaging than in-phase loading $\Rightarrow$ Be carefull when extrapolating uniaxial fatigue characteristics to multiaxial fatigue

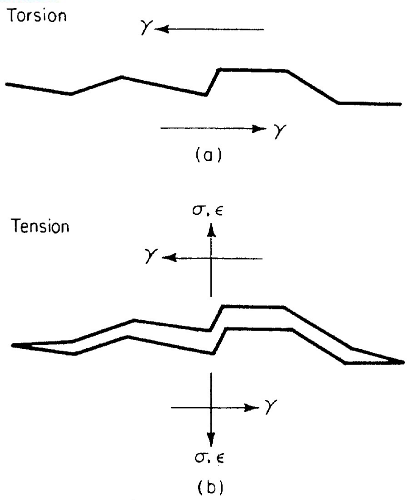
Critical Plane Approach

- Fatigue cracks initiate on planes of maximum shear
- Brown&Millar
  - Critical Plane approach considered the maximum shear strain plane and the tensile or normal strain acting on it

$$\frac{\Delta\gamma}{2} + \Delta\varepsilon_n = C$$

$\frac{\Delta\gamma}{2}$	Is the shear strain amplitude on the maximum shear strain plane
$\Delta\varepsilon_n$	Is the tensile normal strain range to this plane
C	Is the material constant

Critical Plane Approach



- See how tensile strain/stress opens the crack faces
  - Friction between crack faces is eliminated
  - Crack tip experiences all of the applied shear load
  - More damaging than the pure torsion case
- Advantage of the critical plane approach is that it relates predicted fatigue life to experimentally observed cracking behaviour

Comparison of Methods - SAE notched shaft program

- Combined bending and torsion tests were done
- The following life predicting theories were used:
  - Principal strain theory
  - von Mises effective strain theory
  - Maximum shear strain theory
  - Brown & Miller theories
- All do an adequate job of predicting the fatigue life for constant amplitude, completely reversed, in-phase, combined bending and torsion
- Further work suggested for out-of-phase testing

Conclusion

- Fatigue life estimates for in-phase loading reasonably good for all techniques presented
- Predictive multiaxial fatigue theories needed for safe, cost-efficient design of many engineering components