Behaviour of welded structures under dynamic loading

Weld fatigue course

Prepared for Universities

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This document is the master document for the discussion of the behaviour of welded structures under dynamic loading.
Software applications
The following software applications were used in the execution of this project:

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Calculation files
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Models
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Input Data Files
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### Output Data Files

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<td>Class notes.docx. The same notes document is used for both days to have all in one document.</td>
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<tr>
<td>MS Excel document with calculations done in class</td>
<td>Class calculations.xlsx. The same Excel document is used for both days to have all in one document.</td>
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List of symbols and abbreviations

\( k_1 \)  Magnification factor for nominal stress ranges to account for secondary bending moments in trusses

\( k_f \)  Stress concentration factor

\( k_s \)  Reduction factor for fatigue stress to account for size effects

\( m \)  Slope of the fatigue curve

\( m_1 \)  Slope of the fatigue curve for stress range above the constant amplitude fatigue limit

\( m_2 \)  Slope of the fatigue curve for stress ranges between the cut-off limit and constant amplitude fatigue limit

\( N_R \)  Design life time expressed as number of cycles related to a constant stress range.

\( Q \)  Characteristic value of a single action [N]

Greek symbols:

\( \beta \)  Geometric stress concentration factor

\( \gamma_{FF} \)  Partial factor for equivalent constant amplitude stress ranges \( \Delta \sigma_E, \Delta \tau_E \)

\( \gamma_{MF} \)  Partial factor for fatigue strength \( \Delta \sigma_C, \Delta \tau_C \)

\( \varepsilon \)  Strain. If there is a subscript it indicates the direction of the strain. For example, \( \varepsilon_x \) is strain in the x-direction.

\( \eta \)  Eta

\( \theta \)  Angle [°]

\( \lambda_i \)  Damage equivalent factors

\( \nu \)  Poisson ratio

\( \Delta \sigma \)  Direct stress range [Pa]

\( \Delta \tau \)  Shear stress range [Pa]

\( \Delta \sigma_{eq} \)  Equivalent stress range for connections in webs of orthotropic decks [Pa]

\( \Delta \sigma_C, \Delta \tau_C \)  Reference value of the fatigue strength at \( N_C = 2 \times 10^6 \) cycles [Pa]

\( \Delta \sigma_D, \Delta \tau_D \)  Fatigue limit for constant amplitude stress range at \( N_D \) cycles [Pa]

\( \Delta \sigma_E, \Delta \tau_E \)  Equivalent constant amplitude stress range related to \( n_{\text{max}} \) [Pa]

\( \Delta \sigma_{E2}, \Delta \tau_{E2} \)  Equivalent constant amplitude stress range related to 2 million cycles [Pa]

\( \Delta \sigma_L, \Delta \tau_L \)  Cut-off limit for stress ranges at \( N_L \) cycles [Pa]

\( \Delta \sigma_C, \text{red} \)  Reduced reference value of the fatigue strength [Pa]

Note, the lists above do not contain symbols that are clearly defined in the section or that are indicated on sketches.
Conversion factors
Comma indicates thousands: 1,000.00

Angle:
1 circle = 360° = 400 grades

Area:
1 ft² = 0.0929 m²
1 m² = 6.452 × 10⁻⁴ m²

Density:
1 lb/ft³ = 16.018 kg/m³
1 slug/ft³ = 515.379 kg/m³

Energy:
1 Btu = 1.055.06 J
1 cal = 4.186 J
1 hp h = 0.7457 kWh
1 erg = 10⁻⁷ J
1 Btu/lb_m = 2.326 ftl/kg
1 Btu/(lb_m°F) = 4.1868 ftl/(kgK)

Force:
1 lb_f = 4.44822 N

Heat flow rate:
1 Btu/s = 1.055.1 W
1 Btu/h = 0.2931 W
1 hK (metric horse power) = 0.735499 kW
1 hp = 0.74570 kW

Heat transfer coefficient:
1 Btu/h°F = 5.678 W/m²K

Length:
1 ft = 0.3048 m
1 in = 25.4 mm
1 mile = 1.6093 km
1 km = 5.280 ft
1 mm = 1.852 km
Nautical Mile (nm) = Sea Mile
Angstrom = 10⁻¹⁰ m

Luminous Flux:
1 candle power = 12.566 lumen
= 0.0188 W

Mass:
1 lb_m = 0.4536 kg
= 16 oz
= 7000 grain
1 grain = 0.0648 g
1 slug = 14.594 kg
1 oz = 28.35 g
1 ton = 1000 kg
1 short ton = 1.1023131 short tons
1 short ton = 907.18474 kg
1 long ton = 1016.0469088 kg
1 troy pound = 12 troy ounces
1 troy ounce = 31.1034768 g
1 pennyweight = 24 grain
1 carat = 0.200 g

Power:
1 Btu/s = 1.055.1 W
1 Btu/h = 0.2931 W
1 hp = 745.7 W
1 refrigeration ton = 3.516 W
1 cooling tower ton = 15,000 Btu/h

Pressure:
1 atm = 101.325 kPa
= 101.325 Pa
1 Pa = 10⁻⁵ Bar
= 1.45 × 10⁻⁴ psi
1 psi = 6.8948 kPa
1 psf = 47.88 Pa
1 mm Hg = 133 Pa

Speed:
1 ft/s = 0.3048 m/s
1 knot = 1.852 km/h

Stress:
1 psi = 6.8948 Pa
1 ksi = 6.8948 MPa

Structural Mechanics:
1 ksi√in = 1.0792 MPa√m

Temperature:
°F = °C × 9/5 + 32
°C = °F - 32
°K = °C + 273.15

Torque and Moment:
1 ft lb = 1.356 Nm

Viscosity (Dynamic)
1 lb/(ft·s) = 1.4879 Pa·s
1 cP = 10⁻⁴ Pa·s
= 0.001 Ns/m²
= 0.01 Poise

Viscosity (Kinematic)
1 cSt = 10⁻⁶ m²/s
= 0.01 St

Volume:
1 ft³ = 0.02832 m³
1 in³ = 1.6387 × 10⁻⁵ m³
1 Gallon (US) = 3.785 l
1 Imp. Gallon (UK) = 4.546 l
1 pt = 0.568 l
1 dm³ = l (Liter)
1. **INTRODUCTION**

This document presents the class notes to understand the behaviour of welded structures under dynamic loading. This section introduces concepts to students whereas the following Module 3.8 focuses on the design of welded structures under dynamic loading. Therefore, this section is more information and introduces basic principles. Module 3.8 introduces application of the concepts by calculation.

2. **STUDY MATERIAL**

The student shall arrange access to the following documents:

1. Slides presented in class.
2. This guide.

3. **TERMINOLOGY**

This section summarises terminology used in the lectures and come from BS EN 1993-1-9 (2005:7:9):

**Fatigue**

The process of initiation and propagation of cracks through a structural part or member due to the action of fluctuating stress.

**Nominal stress**

A stress in the parent material or in a weld adjacent to a potential crack location calculated in accordance with elastic theory excluding all stress concentration effects. The nominal stress can be a direct stress, shear stress, principal stress or equivalent stress.

**Modified nominal stress**

A nominal stress multiplied by an appropriate stress concentration factor $k_f$ to allow for a geometric discontinuity that has not been taken into account in the classification of a particular constructional detail.

**Geometric stress = hot spot stress**

The maximum principal stress in the parent material adjacent to the weld toe, taking into account stress concentration effects due to the overall geometry of a particular constructional detail. Local stress concentration effects e.g. from the weld profile shape (which is already included in the detail categories in Appendix B of BS EN 1993-1-9 for the geometric stress method) need not to be considered.

**Residual stress**

Residual stress is a permanent state of stress in a structure that is in equilibrium and is independent of the applied actions. Residual stresses can arise from rolling stresses, cutting processes, welding shrinkage or lack of fit between members or from any loading event that causes yielding of part of the structure.

**Loading event**

A defined loading sequence applied to the structure and giving rise to a stress history, which is normally repeated a defined number of times in the life of the structure.

**Stress history**

A record or a calculation of the stress variation at a particular point in a structure during a loading event.
Rainflow method
Particular cycle counting method of producing a stress-range spectrum from a given stress history.

Reservoir method
Particular cycle counting method of producing a stress-range spectrum from a given stress history.

Stress range
Algebraic difference between the two extremes of a particular stress cycle derived from a stress history.

Stress-range spectrum
Histogram of the number of occurrences for all stress ranges of different magnitudes recorded or calculated for a particular loading event.

Design spectrum
The total of all stress-range spectra in the design life of a structure relevant to the fatigue assessment.

Design life
The reference period of time for which a structure is required to perform safely with an acceptable probability that failure by fatigue cracking will occur.

Fatigue life
The predicted period of time to cause fatigue failure under the application of the design spectrum.

Miner’s summation
A linear cumulative damage calculation based on the Palmgren-Miner rule.

Equivalent constant amplitude stress range
The constant amplitude stress range that would result in the same fatigue life as for the design spectrum, when the comparison is based on a Miner’s summation.

Fatigue loading
A set of action parameters based on typical loading events described by the positions of loads, their magnitudes, frequencies of occurrence, sequence and relative phasing.

Equivalent constant amplitude fatigue loading
Simplified constant amplitude loading causing the same fatigue damage effects as a series of actual variable amplitude loading events.

Fatigue strength curve
The quantitative relationships between the stress range and number of stress cycles to fatigue failure, used for the fatigue assessment of a particular category of structural detail. The fatigue strengths given in BS EN 1993-1-9 are lower bound values based on the evaluation of fatigue tests with large scale test specimens in accordance with EN 1990 – Annex D.

Detail category
The numerical designation given to a particular detail for a given direction of stress fluctuation, in order to indicate which fatigue strength curve is applicable for the fatigue assessment (The detail category number indicates the reference fatigue strength $\Delta \sigma_C$ in MPa).
Constant amplitude fatigue limit
The limiting direct or shear stress range value below which no fatigue damage will occur in tests under constant amplitude stress conditions. Under variable amplitude conditions all stress range have to be below this limit for no fatigue damage to occur.

Cut-off limit
The limit below which stress ranges of the design spectrum do not contribute to the calculated cumulative damage.

Endurance
The life to failure expressed in cycles, under the action of a constant amplitude stress history.

Reference fatigue strength
The constant amplitude stress range $\Delta \sigma_c$ for a particular detail category for an endurance of $N = 1 \times 10^6$ cycles
4. BEHAVIOUR OF WELDED STRUCTURES UNDER VARIABLE AMPLITUDE LOADING

4.1. Objective
Understand in detail:
1. The development of fatigue
2. Calculation of load cycles
3. Influence of notches and their avoidance

4.2. Scope
The following material will be covered:
1. Types and variables of cyclic loading
2. Statistical stress analysis on real structures
3. S-N diagrams
4. Stress collective
5. Fatigue strength
6. Effect of mean stress including residual stresses
7. Effect of stress range
8. Stress distribution
9. Influence of notches
10. Influence of weld imperfections
11. Weld fatigue improvement techniques
   a. Surface protection
   b. Needle peening
   c. TIG dressing
   d. Burr grinding
   e. Hammering
   f. Stress relieving
12. Standards, ISO, CEN and National
13. Palmgren-Miner rule
14. Classification of weld joints

4.3. Outcomes
After completion of this section you will be able to:
1. Draw and interpret an S-N diagram.
2. Explain fully the methods of counting load cycles.
3. Calculate the stress ratio.
4. Detail the influence of notches and weld defects.
5. Explain fully the methods for improving fatigue performance.
Static loading = use static failure criteria:
- Maximum principal stress theory
- Maximum shear stress theory
- Von Mises theory
- Mohr’s failure theory
- Maximum normal strain theory
- Total strain energy theory
- Fracture for cracked members
- Standards

Variable amplitude loading

A defect in the material grows to a detectable size where after it propagates to fracture.

Fatigue analysis
Step 1: Do peak-valley reduction (find extremes) of the signal to determine turning points
Step 2: Rainflow counting
Step 3: Present in stress-range histogram. The outcome is: $\Delta \sigma_i, n_i$
Step 4: Model the S-N curve and calculate endurance for each stress range
Step 5: Calculate damage for each stress range
Step 6: Calculated total damage according to the Palmgren-Miner rule

Fracture mechanics
Step 1: Do peak-valley reduction (find extremes) of the signal to determine turning points
Step 2: Calculate positive stress intensity change for each reversal (half cycle) $\Delta K = \Delta \sigma \beta \sqrt{\pi a}$. Only use positive values
Step 3: Calculate crack propagation from the Paris rule: $\frac{da}{dN} = C \Delta K^m$
Step 4: Accumulate crack size to the critical crack size, $a_{cri}$
5. **FATIGUE S-N CURVES AND DAMAGE**

The objective of this section is to introduce S-N fatigue curves to the student and enable the student to carry out fatigue calculations for constant amplitude loads.

<table>
<thead>
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<th>Presentation used in class:</th>
<th>S-N curves and damage</th>
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### 5.1. Constant amplitude alternating stress parameters

The purpose of this section is to evaluate the behaviour of weld detail under cyclic loading. The theory of BS EN 1993-1-9 and IIW Bulletin 520 will be applied.

Fatigue failure occurs when a material is subject to repeated cyclic loading as in the case of a train wheel, ball mill, shafts, etc. Fatigue failure involves crack initiation/formation, growth and final fracture. Fatigue failure can occur at stresses well below the yield or ultimate tensile strength of the material and is affected by alternating stresses, any mean stresses, surface finish, environmental conditions, and presence of notches and flaws. An example of an alternating stress range with the applicable stresses can be seen in Figure 1.

![Figure 1: An example of a constant amplitude alternating stress signal](image)

The stress range is calculated using the following equation:

\[
\Delta \sigma = \sigma_{max} - \sigma_{min} \tag{1}
\]

The stress amplitude is calculated using the following equation:

\[
\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} \tag{2}
\]

The mean stress is calculated using the following equation:

\[
\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \tag{3}
\]

The stress ratio is:

\[
R = \frac{\sigma_{min}}{\sigma_{max}} \tag{4}
\]

The amplitude ratio is:

\[
A = \frac{\sigma_a}{\sigma_m} \tag{5}
\]

Where:

\(\Delta \sigma\) Stress range [MPa]
\[ \sigma_a \text{ Stress amplitude [MPa]} \]
\[ \sigma_m \text{ Mean stress [MPa]} \]
\[ \sigma_{\text{max}} \text{ Maximum stress [MPa]} \]
\[ \sigma_{\text{min}} \text{ Minimum stress [MPa]} \]

From these it is clear to see that for:

- Completely reversed signals:
  \[ R = -1; A = \infty \]
- Signal from zero to a maximum:
  \[ R = 0; A = 1 \]
- Signal from zero to a minimum:
  \[ R = \infty; A = -1. \]

5.2. S-N curve for steel

The S-N curve for 300W steel shown in Figure 2 (for 50 % probability of failure) is modelled by the equation:

\[ S^m N = C \quad (6) \]

Where

- \( S \) is the completely reversed stress amplitude, in MPa

For steel, the limits of the S-N curve are:

- a life of 1 000 cycles at completely reversed stress amplitude \( 0.9 f_{ut} \), and,
- endurance limit at \( 10^6 \) cycles at completely reversed stress amplitude \( 0.5 f_{ut} \).

These ratios are available for other materials as well and will be discussed in the applicable parts of the course.

The S-N curve constructed from these parameters predicts an endurance, \( N \), with a 70% confidence level of a 50% probability of survival.

With the exponent \( m \) and endurance limit \( S_e \), or any other stress amplitude endurance combination known, the S-N curve can be constructed for endurance as follows:

\[ N_R = \begin{cases} \left( \frac{S_1}{S_a} \right)^m & S_R \geq S_e \\ \infty & S_R < S_e \end{cases} \]

This equation is used with the Palmgren Miner rule to calculate damage for any probability of failure and for any stress spectrum.
Figure 2: Completely reversed S-N curve for structural steel with yield strength 300 MPa for a confidence level of 70% of 50% probability of survival for N.
5.2.1. Example

Construct the S-N curve for a material with completely reversed stress amplitude $S_1 = 500 \text{ MPa}$ at $N_1 = 10^3$ and endurance limit $S_e = 150 \text{ MPa}$ at $N_e = 10^6$ cycles.

Solution

Two points are given on the S-N curve, for which the following equation applies:

$$S_1^m N_1 = S_2^m N_2$$ \hspace{1cm} (7)

The exponent, $m$, can be calculated as follows:

$$m = \frac{\log \left( \frac{N_2}{N_1} \right)}{\log \left( \frac{S_1}{S_2} \right)} \hspace{1cm} (8)$$

Now, the endurance, $N_R$ at any completely reversed stress amplitude, $S_a$ is as follows

$$N_R = \left\{ \begin{array}{ll}
(S_1/S_a)^m N_1 & 0.9f_{ut} \geq S_R \geq S_e \\
\infty & S_R < S_e
\end{array} \right. \hspace{1cm} (9)$$

5.3. Non-ferrous alloys

The pseudo-endurance limit is specified as the stress value at 500 million cycles.

5.4. Fatigue ratio

The fatigue ratio is the ratio of endurance limit to ultimate tensile strength:

$$f_r = \frac{S_e}{f_{ut}}$$

For steel $f_r$ varies between 0.35 and 0.6. Most steels with $f_{ut} < 1400 \text{ MPa}$ have a fatigue ratio of 0.5.

5.4.1. Endurance limit and surface hardness

As function of surface hardness of material:

$$S_e = \left\{ \begin{array}{ll}
0.25BHN \text{ ksi} & \text{for } BHN \leq 400 \\
100 \text{ ksi} & \text{for } BHN > 400
\end{array} \right.$$

5.4.2. Endurance limit and ultimate tensile strength of steel and cast steel

In terms of ultimate tensile strength:

Steel

$$S_e = \left\{ \begin{array}{ll}
0.5S_{ut} & \text{for } S_{ut} \leq 200 \text{ ksi (1400 MPa)} \\
(100 \text{ ksi (700 MPa)}) & \text{for } S_{ut} > 200 \text{ ksi (1400 MPa)}
\end{array} \right.$$ 

Cast Iron + Cast Steels:

$$S_e = \left\{ \begin{array}{ll}
0.45S_{ut} & \text{for } S_{ut} \leq 600 \text{ MPa} \\
(275 \text{ MPa}) & \text{for } S_{ut} > 600 \text{ MPa}
\end{array} \right.$$ 

5.4.3. Other materials

Deriving constants for other materials:

- Not so simple because of potential non-linear S-N characteristics
- Linear approximation used in most applications due to empirical data used in analysis – statistical errors exists already
5.5. **Typical EN 1993-1-9 \( S_r - N \) curve**

The fatigue curves used in EN 1993-1-9 standard has the form shown in Figure 3. Please populate the curve with information supplied in the slides.

\[
\text{\( S_r - N \) for characteristic strength = 100.0 MPa}
\]

![Fatigue Curve](image)

**Figure 3**: Typical EN 1993-1-9 \( S_r - N \) curve for characteristic strength \( \Delta \sigma_c = 100 \text{ MPa} \)

5.5.1. **Using the idea**

Calculate the constant amplitude fatigue limit and the cut-off limit for a detail category 100, i.e., the characteristic strength is \( \Delta \sigma_c = 100 \text{ MPa} \).

**Solution**

Make your own notes from information supplied in class.
5.6. Damage modelling and summing

5.6.1. Linear damage rule – Palmgren-Miner’s rule

The linear Palmgren-Miner’s damage rule states that the damage due to a stress amplitude is equal to the number of cycles that the stress amplitude is applied divided by the endurance (number of cycles to crack initiation) at that stress amplitude.

\[ D_i = \frac{n_i}{N_i} \]  

(10)

If there are more than one stress amplitude applied to the component, then the total damage is the sum of the damages due to the individual stress conditions, as shown in Figure 4.

\[ D_T = \sum_{i=1}^{k} \frac{n_i}{N_i} \]  

(11)

Test results showed that crack initiation occurs when the total damage range between 0.5 and 2.0. For calculation purposes, failure will be assumed when \( D_T = 1.0 \).

\[ 0.5 \leq D_T \leq 2.0 \]

![Figure 4: Linear cumulative damage - Palmgren-Miner’s rule on an \( S_r – N \) curve](image)

5.6.2. Shortcomings of the linear damage rule

- It does not consider sequence effects
- It is amplitude independent
  - Predicts that the rate of damage accumulation is independent of stress level. Observed behaviour show that at high strain amplitudes cracks will initiate in a few cycles, whereas at low strain amplitudes almost all of the life is spent initiating a crack.

Non-linear damage theories
- Practical problems
- Require material and shaping constants which must be determined experimentally
- Sequence effects must be tested for
  - Cannot be guaranteed that these methods will be more accurate than Miner's rule

5.6.3. **Summary**
- Use the Palmgren-Miner rule
- Non-linear techniques not significantly more accurate
- Damage summation techniques must account for load sequence effects (mean stress, residual stress) ⇒ use **strain life** to account for initially high stresses
6. **NOTCH STRESS ASSESSMENT OF WELD DETAIL**

This section presents an introduction to explain the stress concentrations caused by notches. The notches at weld toes and other weld imperfections are the predominant reason for reduced fatigue strength of weld detail. The focus of this section is on the stress distribution at weld toes and roots.

<table>
<thead>
<tr>
<th>Presentation used in class:</th>
<th>Notch stress assessment of weld detail</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filename: Investmech - Structural Integrity (Notch effects of welds) R0.0</td>
<td></td>
</tr>
</tbody>
</table>

6.1. **Principle of the notch stress approach**

Weld toes, geometric changes, cuts, holes, etc result in an increase in stress. The notch at a weld toe result in higher stress concentrations where cracks can initiate. In the notch stress approach, the stress in the vicinity of the weld toe is calculated for fatigue purposes. When the stress is predicted with finite element analysis, no plastic constitutive material models may be used. Linear elastic constitutive models shall be used to enable usage of the fatigue curves given by the EN 1993-1-9 and BS 7608 and other standards. The following hypotheses were applicable for notch stress analysis:

- Stress gradient approach (Siebel & Stieler, 1955)
- Critical distance approach (Peterson, 1959)
- Highly stressed volume approach (Kuguel, 1961; Sonsino, 1994 & 1995)

The last three methods are used for weld notch stress analysis.

6.2. **Fictitious notch rounding – effective notch stress approach**

It is not possible to model an infinitely sharp notch by finite element analysis methods, because of the requirement to have a certain number of nodes on a curve to ensure proper reconstruction thereof. For finite element analysis, infinitely sharp notches are fictitiously rounded as shown in Figure 5. This method is known as the effective notch stress approach.

According to Neuber (Fricke, 2010):

\[
\rho_f = \rho + s \rho^* 
\]  

(12)

Where:

- \(\rho\) is the actual notch radius [m]
- \(\rho^*\) is the substitute micro-structural length [m]
- \(s\) is a factor for stress multiaxiality & strength criterion

For welded joints, \(s = 2.5\) for plane strain conditions at the roots of sharp notches, combined with the von Mises strength criterion.
Figure 5: Notch stress application

The substitute micro-structural length for different materials is shown in Figure 6 from which the following can be concluded:

1. For low strength steel, $\rho^* = 0.4$.
   a. This results in an increase of $2.5 \rho^* = 1 \text{ mm}$ increase in radius.
   b. For the worst case of a notch with actual radius 0, the fictitious radius is: $\rho_f = 0 + 2.5 \times 4.0 = 1.0 \text{ mm}$.

6.2.1. Critical distance approach

The critical distance approach employs material constants and notch radius to reduce the elastic stress concentration factor $K_t$ to the fatigue notch factor $K_f$. This will be discussed in detail as part of stress life analysis later in the course.
6.3. **Simple design S-N curve according to IIW Bulletin 520**
The slide below summarises the fatigue curve with the applicable characteristic values in the table. This enables efficient estimation of fatigue life when notches are modelled.

6.4. **Design S-N curve**
Both BS 7608 and IIW Bulletin 520 has fatigue curves for use with stress calculated from a notch stress approach with a specified effective notch radius.

**Table 1: Characteristic fatigue strength for welds of different materials based on effective notch stress with** \( r_{\text{ref}} = 1 \text{ mm} \) (maximum principal stress)

<table>
<thead>
<tr>
<th>Material</th>
<th>Characteristics strength ( (p_x = 97.7%, N = 2 \times 10^6) )</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium alloys</td>
<td>FAT 71</td>
<td>Morgenstern et al. (2004)</td>
</tr>
<tr>
<td>Magnesium</td>
<td>FAT 28</td>
<td>Karakas et al. (2007)</td>
</tr>
</tbody>
</table>

Source: (Fricke, 2010, p. 18)

The equation for the S-N curve above the constant amplitude fatigue limit of steel is given as:

\[
C = \Delta \sigma^m N \\
C = FAT^m \cdot 2 \times 10^6 \\
m = 3
\]  (13)

Later in the course more on this.

6.5. **Applied to a cruciform joint**
The slide below shows the finite element model with boundary conditions of a cruciform joint with:

1. The top weld left end sharp.
2. The top weld right end with notch radius 1.0 mm.
3. Complete joint penetration for the bottom weld.

Remember to always consider the direction of principal stresses in calculations.
Cruciform joint

Material is steel with:
\[ E = 210 \text{ GPa} \]
\[ v = 0.3 \]
\[ \rho = 7850 \text{ kg/m}^3 \]

Fillet weld at top
Complete joint penetration weld at bottom

Weld toes have radius of 1 mm
Weld root has hole with diameter 2 mm

The edges on the horizontal section was constrained for translation in all directions
Edge loads of 100 MPa pressure were applied to the top and bottom sections

16 mm cruciform joint with 11.2 mm weld

Maximum principal stress in the 16 mm cruciform joint made with 11.2 mm fillet and complete joint penetration welds

Note the stress concentration caused by the geometrical changes at the weld toes and root
6.6. References
Fricke, W. 2010. Guideline for the Fatigue Assessment by Notch Stress Analysis for Welded Structures. *International Institute of Welding, IIW-Doc. XIII-2240r2-08/XV-1289r2-08*
7. STATIC FAILURE THEORIES

The objective of this section is to provide revision of static failure theories used as acceptance criteria in design. Investmech assumes that these sections have been discussed in detail in applications by other lecturers and will not be repeated in detail.

This section covers the following:

- Calculation of principal stresses
- Static failure theories
- Buckling
- Variable loading

Always remember to include the design class of the structure in failure theories. For example, multiple load paths classify a structure as fail-safe, and other structures are designed for safe-life. In safe-life applications, damage tolerant design is essential.

<table>
<thead>
<tr>
<th>Presentation used in class:</th>
<th>Static Failure Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filename: Investmech - Structural Integrity (Static Failure Theories) R0.0</td>
<td></td>
</tr>
</tbody>
</table>

7.1. Principal stress

Stress in any direction \( l, m \) and \( n \) can be calculated for a multi-axial stress state using the following equation:

\[
\sigma_n = l^2 \sigma_x + m^2 \sigma_y + n^2 \sigma_z + 2[lt \tau_{xy} + nlt \tau_{xz} + mnt \tau_{yz}]
\]  \((14)\)

Where, \( l, m \) and \( n \) are the direction cosines of the unit vector directing in the direction in which the stress is required. The stress matrix is given as follows:

\[
\bar{\sigma} = \begin{bmatrix}
\sigma_x & -\sigma_{xy} & -\sigma_{xz} \\
-\sigma_{yx} & \sigma_y & -\sigma_{yz} \\
-\sigma_{zx} & -\sigma_{zy} & \sigma_z
\end{bmatrix}
\]  \((15)\)

There exists a direction where the shear stress components disappear and only normal stress remain. These directions and resulting normal stress are the principal axes and principal stresses respectively. The principal stresses are calculated from the eigenvalues of the stress matrix as follows:

\[
\begin{vmatrix}
\sigma - \sigma_x & -\sigma_{xy} & -\sigma_{xz} \\
-\sigma_{yx} & \sigma - \sigma_y & -\sigma_{yz} \\
-\sigma_{zx} & -\sigma_{zy} & \sigma - \sigma_z
\end{vmatrix} = 0
\]

\[
\sigma^3 - \sigma^2(\sigma_x + \sigma_y + \sigma_z) + \sigma(\sigma_x \sigma_y + \sigma_y \sigma_z + \sigma_z \sigma_x - \sigma^2_{xy} - \sigma^2_{xz} - \sigma^2_{yz})
- (\sigma_x \sigma_y \sigma_z + 2\sigma_{xy} \sigma_{xz} \sigma_{yx} - \sigma_x \sigma^2_{yz} - \sigma_y \sigma^2_{xz} - \sigma_z \sigma^2_{xy})
\]  \((16)\)

The eigenvalues, or principal stresses can be easily calculated in PC Matlab using the \( E = \text{eig}(X) \) command, where \( X \) is the stress matrix containing the numerical stress values in the sign convention shown in Equation 15.

For example, say the multi-axial stresses are as follows:

\[
\begin{align*}
\sigma_x &= 600 \text{ MPa} \\
\sigma_y &= 0 \text{ MPa} \\
\sigma_z &= 0 \text{ MPa} \\
\sigma_{xy} &= 400 \text{ MPa} \\
\sigma_{xz} &= 0 \text{ MPa} \\
\sigma_{yz} &= 0 \text{ MPa}
\end{align*}
\]

Then the matrix \( X \) is:

\[
X = [600 - 400 0; -400 0 0; 0 0 0]
\]

Performing this calculation in Matlab gives:

\[
E = \begin{bmatrix}
-200 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 800
\end{bmatrix}
\]  \((17)\)
For a two dimensional problem, the maximum ($\sigma_1$) and minimum ($\sigma_2$) principal stresses can be calculated using the following equations:

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \quad (18)$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \quad (19)$$

The principal stresses are then ordered from large to small with $\sigma_1, \sigma_2$ and $\sigma_3$ the maximum, intermediary and minimum principal stress.

7.2. Principal axes

The principal axes are easily determined by PC Matlab using the command: [V,D]=eig(x). The matrix D contains the eigenvalues and the columns of matrix V represent the eigenvectors in the coordinate system used to compile the stress matrix. For the previous values for X is used, this calculation gives:

```matlab
>> [V,D]=eig(X)
V =
    -0.4472    0  -0.8944
   -0.8944    0    0.4472
     0    1.0000    0
D =
   -200   0   0
     0   0   0
     0   0  800
```

In this case the maximum principal stress is $\sigma_1 = 800$ MPa in the direction $\vec{r}_1 = -0.8944\hat{i} + 0.4472\hat{j} + 0\hat{k}$. The minimum principal stress is $\sigma_3 = -200$ MPa in the direction $\vec{r}_3 = -0.4472\hat{i} - 0.8944\hat{j} + 0\hat{k}$.

In the following example the calculation is repeated for an obvious direction of the principal axes:

```matlab
>> X=[600 0 0; 0 200 0; 0 0 100];
>> [V,D]=eig(X)
V =
     0    0    1
     0    1    0
     1    0    0
D =
    100   0   0
     0  200   0
     0   0  600
```

In this case the maximum principal stress is $\sigma_1 = 600$ MPa and the minimum stress is $\sigma_3 = 100$ MPa. The direction of the principal axes is clear in this case. For example, $\vec{r}_3 = 1\hat{i} + 0\hat{j} + 0\hat{k}$ for the stress $\sigma_3 = 600$ MPa. That is the, the third column of V aligns with the third column (and row) where $D = 600$ MPa.

These mathematics are carried out on the multi-axial stress state at a point to find the principal stresses and their associated directions relative to the coordinate system used to construct the stress matrix.

7.3. Failure theories

7.3.1. Maximum normal stress

In this case the maximum principal stresses are compared against the material resistances. The maximum and minimum principal stress is $\sigma_1$ and $\sigma_3$ respectively, where $\sigma_1$ and $\sigma_3$ are not necessarily in tension or compression. Consider the unique situation of hydrostatic stress where $\sigma_1 = \sigma_2 = \sigma_3$. Acceptance criteria are normally the material's yield or ultimate strength or a factor thereof. For example, the partial factor for strength in SANS 10162-1 is $\phi = 0.9$ for structural steel sections.
Where both tensile and compressive stresses are present, tensile and compressive material resistance must be used.

### 7.3.2. Maximum shear stress

The maximum shear stress is:

\[
\tau_{\text{max}} = \frac{\sigma_1 - \sigma_3}{2} \quad (20)
\]

Failure occurs when the maximum shear stress exceeds the fracture stress, \(\sigma_f\):

\[
\tau_{\text{max}} \geq \frac{\sigma_f}{2} \\
\text{or} \\
|\sigma_1 - \sigma_2| \geq \sigma_f \\
|\sigma_1 - \sigma_3| \geq \sigma_f \\
|\sigma_3 - \sigma_2| \geq \sigma_f \quad (21)
\]

Normally the yield strength, ultimate strength or factors thereof are used for the fracture strength.

### 7.3.3. Von Mises failure theory

The Von Mises theory is used to predict yielding of materials under multi-axial stress state. Failure occurs when:

\[
(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2 \geq 2f_y^2 \quad (22)
\]

Where:

- \(\sigma_{1,2,3}\) Principal stresses [MPa]
- \(f_y\) Yield strength of the applicable material [MPa]

The finite element analysis programs automatically calculate the principal stresses as well as the equivalent Von Mises stress at each node on the mesh.

Figure 7 shows a graph of the static failure theories.

![Static failure theories, \(f_y = 300\) MPa](image)

Source: Investmech algorithm staticfailurestheories.m

**Figure 7:** Comparison of failure theories
7.3.4. **Octahedral shear stress theory**

The octahedral shear stress predicts failure exactly as the von Mises failure theory given above.

\[
\tau_{oct} = \frac{1}{3} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2} \geq \sqrt{\frac{2}{3}} f_y
\]  

(23)

7.3.5. **Mohr’s failure theory**

The Mohr’s failure theory is also known as the Coulomb-Mohr criterion or the internal friction theory, and, is based on the Mohr circle shown below. The angle of radius \( R \) with the horizontal is \( 2\theta \) and is used to find the plane of maximum stress. This indicates that if only shear stress is applied, the maximum principal stress will be on an angle of 45° because \( 2\theta = 90° \).

<table>
<thead>
<tr>
<th>Case</th>
<th>Principal stresses</th>
<th>Acceptance criterion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Both in tension</td>
<td>( \sigma_1 &gt; 0, \sigma_2 &gt; 0 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \sigma_1 &lt; \sigma_c, \sigma_2 &lt; \sigma_c )</td>
</tr>
<tr>
<td>2</td>
<td>Both in compression</td>
<td>( \sigma_1 &lt; 0, \sigma_2 &lt; 0 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \sigma_1 &gt; -\sigma_c, \sigma_2 &gt; -\sigma_c )</td>
</tr>
<tr>
<td>3</td>
<td>( \sigma_1 ) in tension, ( \sigma_2 ) in compression</td>
<td>( \sigma_1 &gt; 0, \sigma_2 &lt; 0 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \frac{\sigma_1}{\sigma_t} + \frac{\sigma_2}{-\sigma_c} &lt; 1 )</td>
</tr>
<tr>
<td>4</td>
<td>( \sigma_1 ) in compression, ( \sigma_2 ) in tension</td>
<td>( \sigma_1 &lt; 0, \sigma_2 &gt; 0 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \frac{\sigma_1}{-\sigma_c} + \frac{\sigma_2}{\sigma_t} &lt; 1 )</td>
</tr>
</tbody>
</table>

Mohr’s versus the maximum stress criterion will be explained in class according to the sketch below.

7.3.6. **These are not all**

Note that there are several other static failure theories applied by Investmech that are not presented here. For example, additional failure theories include:

- Maximum normal strain theory (St. Venant’s theory)
- Total strain energy theory (Beltrami theory)
- Etc.

7.3.7. **Which static theory can be used**

Research showed that for:

1. Ductile material types (> 5 % elongation at break):
   a. The maximum shear stress criterion and the von Mises criterion are accurate static failure theories
2. Brittle material types (≤ 5% elongation at break):
   a. The maximum normal stress criterion and the Mohr’s theory provide accurate results. Investmech always applies the maximum principal stress theory and the von Mises theory for static design. The use of the von Mises theory without the maximum normal stress theory could result in wrong answers because of the way in which the von Mises equation eliminates hydrostatic stress conditions.

For crashworthiness, furnace design, or other applications where there is substantial plastic deformation, the maximum engineering strain and true principal strains are used.

7.4. Buckling
Columns under compressive loading have a length dependent critical load where the section will fail under buckling. A vertical column hinged at the bottom has a critical load of:

\[
P_{cr} = \frac{\pi^2 EI}{L^2}
\]

Different equations exist for the different boundary conditions and should be applied according to the constraints relevant to the practical problem.

7.5. Non stress-based criteria
There is several other mission profile driven criteria that also need to be considered in the design of structures. For example:

- Success of parts not necessarily determined by strength
  - Stiffness, vibrational characteristics, fatigue resistance, creep resistance
- Examples
  - Rigidity in automotive vehicles
  - Weight reduction in bicycles
  - Patio deck – stiffness to prevent excessive deformation

7.6. Conclusion
- Always look at all failure criteria, or at least two:
  - The one that prevails first will be the mode in which failure will occur
- For furnaces, the maximum total strain of 20% is used to quantify the number of heat-up cycles
- Fatigue analysis is concerned with the calculation of damage to the structure and is the life until a detectable crack initiates.
- Fracture Mechanics determines failure during the crack growth phase.

8. VARIABLE AMPLITUDE LOADING
The objective of this section is to provide an understanding of the analysis of dynamic loads on structures.

The scope of the work includes:
1. Types of loading.
2. Statistical stress analysis on real structures.
4. Mean stress calculation and its effect

After completion of this section you will be able to:
1. Describe methods of counting load cycles.
2. Calculate stress ration and other statistical parameters.

<table>
<thead>
<tr>
<th>Presentation used in class:</th>
<th>Variable Amplitude Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filename: Investmech - Structural Integrity (Variable Amplitude Loading) R0.0</td>
<td></td>
</tr>
</tbody>
</table>
8.1. Types of loading

Loads can be tensile loads, compressive loads and/or shear loads (torsion) that produce stress and
deformation of the component.

Loads can be divided into the following:

- **Static load**
  - Do not change over time
  - SANS 10160-1 and other standard refer to static loads as dead- or permanent loads or actions.

- **Quasi-static load**
  - Applied at rate lower than lowest natural period of the structure
  - Investmech uses a load application period of 5 times the lowest natural period as a quasistatic load. The is equal to a frequency of 1/5th the lowest natural frequency.

- **Dynamic loads**
  - Load application period shorter than 5 × lowest natural period, and typically cause force effects due to natural modes
  - Shock load
    - Impact load period significantly shorter than the natural period

8.2. Causes for loads on structures

There are many causes for variable amplitude loads on structures of which the following is a typical list:

- **Thermal**
  - Normally quasi-static
  - Can be deterministic

- **Process activities**
  - Random, but steady-state

- **Wind loads**

- **Cavitation**

- **Fluid-structural interactions**
  - Water-hammer
  - Tidal – normally quasi-static of nature
  - Waves

- **Mechanical loads**
  - unbalance, misalignment, screen suspension, crusher supports, impact hammers, etc.

- **Human interactions**
  - Dropping objects, explosions, crushes, etc.

- **Cluster events** (Storms, process in reactors, accidents, derailing of railway vehicles, etc.)

- **Etc.**

8.3. Loads and stress-strain

Loads are converted to stress. The stress can be determined in typically the following ways:

- **Elementary equations:**
  - Normal stress: \( \sigma_n = \frac{F}{A} \)
  - Bending stress: \( \sigma_b = \frac{M_y x}{I_{xx}} - \frac{M_x y}{I_{yy}} \)
  - Shear stress (Torsion): \( \tau = \frac{VQ}{A} + \frac{\nu r}{J} \), note the directions!

- **Finite element analysis**

- **Strain measurement and conversion to stress**
8.4. Classification of signal types

Classification of signal types

8.5. Statistical parameters

8.5.1. Peak value

This is the maximum absolute value of the signal \( V_p = \max(|signal(t)|) \). In Matlab the command is:

\[
V_p = \text{max(abs(signal))};
\]

In most cases the peak value will refer to the maximum value of the signal, and not of the absolute value of the signal. Then it is the highest point on a plot of the signal.

8.5.2. Peak-to-peak value

The peak-to-peak value is the same as the range of the signal and is the difference between the maximum and the minimum value:

\[
V_{p-p} = \max(signal(t)) - \min(signal(t))
\]

8.5.3. Mean

\[
\mu = \lim_{T \to \infty} \frac{1}{T} \int_0^T f(t) \, dt
\]

\[
= \frac{1}{N} \sum_{i=1}^{N} f(i)
\]

- For a random signal the mean is zero
8.5.1. Root-mean-square

\[
RMS = \sqrt{\lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} f(t)^2 dt} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} f(i)^2}
\]

- Gives the intensity of the data which is an indication of the energy
- This is an Overall Value, that is, one value that describes the characteristic of all the values
- With band-pass filter will give narrow-band intensity

8.5.2. Crest factor

\[
CF = \frac{\text{Peak of the signal}}{RMS}
\]

- For a pure sine wave \( \text{cf} = 2^{(1/2)} \)
- A \( \text{cf} > 3 \) indicates on irregularities in the signal
- The \( \text{cf} \) is not monotone
  - Will not necessarily increase with an increase in RMS
- Used to describe the “peakiness” of a function/signal

8.5.3. Variance and standard deviation

\[
\sigma^2 = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} (f(t) - \mu)^2 dt
\]

\[
= \frac{1}{N} \sum_{i=1}^{N} (f(i) - \mu)^2
\]

Variance = (standard deviation)\(^2 = \sigma^2\)
The standard deviation quantifies the distribution of data points around the mean.

8.5.4. Kurtosis

\[
KU = \frac{1}{\sigma^4} \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} (f(t) - \mu)^4 dt
\]

\[
= \frac{1}{\sigma^4} \times \frac{1}{N} \sum_{i=1}^{N} (f(i) - \mu)^4
\]

- The kurtosis is not monotone
- Describe the peakiness of a signal
- For sine wave \( \text{KU} = 2 \)
- For a random signal \( \text{KU} = 1.5 \)

8.5.5. Example

Space allowed for class problem on time domain analysis

Sample record given: -1; -0.5; 0; 0.5; 1; 0.75; 0.5; 0.2; -0.2

Peak-to-peak value
8.6. Presentation of stress data

It is important that stress data is accurate. Stress data for fatigue calculation is normally presented as the stress ranges with the number of occurrences in the signal. That is, a table with the one column containing stress ranges and the next column the number of times that the specific stress range occurred in the signal. Stress data can also be presented as a spectrum using FFT calculations. This is only accurate for steady-state responses. In most cases the actual stress signal is not known exactly, but, can be accurately described by the mean and standard deviation.

8.7. Stress histories

The sketch below provides a list of typical stress histories.
8.8. Obtaining stress spectra for fatigue calculations

With computing power today, measured strains converted to stress can be used directly to calculate the stress spectrum for the measurement point.

Load spectra for stress measured or from transient analysis:

- PSD or Spectral analysis
  - Note, the results is for a specific period and must be scaled!

- Counting methods
  - Peak counting, Mean-crossing peak count, Range pair count, Range-pair-mean count, Rainflow count, Reservoir counting method
  - The counting method must produce the correct crack initiation and growth result
  - Counting method must detect peak, mean, minimum, and maximum of signal
  - The results are presented in a histogram for $\Delta \sigma_R$ and $n_R$
  - Note, the results is also only for the duration of the signal measured and must be scaled

8.9. Exceedence diagrams

This is beyond the scope of this course and is only presented in class as an example.

The diagram shows how many times a certain stress or load level is exceeded. The method makes use of peaks and troughs.

These diagrams have same general shape with deviations on the “straightness” of the lines.
Standard spectra:
- Exceedence diagrams from a great number of structures are available and can be used in calculations

8.9.1. Errors and effects
The following is a list of actions carried out on counted cycles that can affect accuracy. However, this is not part of the scope of this course and is only listed for information.

1. Clipping
   a. Counting error because a certain level was not exceeded.

2. Truncation:
   a. Truncation is the process to reduce the number of small cycles to save on computing time
   b. Process reconstructs the lower step
   c. Requires judgement and evaluation of its effect
   d. The larger the number of stress levels, the less the effect of truncation

3. Different periods of severity
   a. For example a vehicle driving on tar and gravel roads. The relative distances are essential to ensure proper design.
8.10. Variable amplitude loading

- Most service loading histories have variable amplitude
- Sometimes stochastic of nature (random probability distribution, may be analysed statistically but cannot be predicted precisely)
- The following aspects need to be addressed:
  - Nature of fatigue damage and how it can be related to load history
  - Damage summation methods
  - Cycle counting techniques to recognise damaging events
  - Crack propagation behaviour under variable amplitude loading
  - How to deal with service load histories
- Fatigue is the tendency of materials to fail due to cracks that initiates and propagates
- Definition of fatigue damage
  - The measurable propagation portion of fatigue
    - Damage is directly related to crack length ⇒ it is observable, measurable
    - Inspection intervals used to monitor crack growth
  - Initiation phase
    - Mechanisms on microscopic level (dislocations, slip bands, micro-cracks, etc.)
    - Only measurable in highly controlled laboratory environment

  ⇒ Most damage summing methods during initiation phase empirical of nature

8.11. Cycle counting methods – discussed

The objective of cycle counting techniques for fatigue analysis is to reduce the data required in analysis. For example, in a strain/stress signal for fatigue analysis, the turning points (extrema) are required, not the data points in between.

8.11.1. Level crossing

Number of times that strain or stress of certain value is crossed. Can be done on the original signal or range levels.
8.11.2. Peak counting

Counts the number of times that a peak is formed in a specified range, 1 in this case. The example below scales down, that is why Point B is indicated to be peak 2. Investmech scales up, so, in this case Peak B would have had value 3. Upscaling is conservative.
8.11.3. Simple-range counting
8.11.4. Rainflow counting
Rainflow counting, giving similar answers than reservoir counting, is the most widely used counting method for stress life analysis. See the following demonstrations of the method:

1. Demonstration done in class.
2. Videos:
   a. https://youtu.be/rI3y4fDXDwM
   b. https://youtu.be/Fvk64Qn9K7E
8.11.5. **Sequence effects**

Strain-time histories may yield very different stress-strain responses, especially in notches where plasticity occurs as shown in Figure 11. Level-crossing, peak-counting and simple range counting do not include sequence effects. If the rainflow counting algorithm is applied in sequence, it does include sequence effects (from-to values in the Markov matrix from which mean, amplitude and range can be calculated), one of the reasons why it is used widely. For these situations, the strain dependent residual strains in the notch need to be modelled, for which strain life principles are used.

![Figure 11: Load sequence effects](image)

8.11.6. **More info on Rainflow counting**

- A number of rainflow counting techniques are in use
- If the strain-time history being analyzed begins and ends at the strain value having the largest magnitude, whether it occurs at a peak or a valley, all of the rainflow counting techniques yield identical results
- Develop the Markov Matrix and find strain amplitudes and mean stress
- Use the Morrow equation to solve the fatigue life at each strain level:
  \[
  \frac{\Delta \varepsilon}{Z} = \frac{\sigma_f - \sigma_o}{E} \left[2N_f\right]^b + \varepsilon_f \left[2N_f\right]^c
  \]
- Calculate cumulative damage from Miner’s rule: \( D = \sum \frac{\varepsilon_f}{N_f} \geq 1 \), note, reversals = half cycles are used in the Morrow equation
- ASTM standard for Rainflow counting in literature
8.12. Crack growth retardation using load sequence effects

Under variable amplitude loading, and the residual stress caused by preceding load cycles, crack propagation depends on the preceding load cycle that can significantly affect the crack growth rate as shown in Figure 12, from which the following may be concluded:

- Single overload causes a decrease in crack growth rate
  - If overload is large enough, crack arrest can occur
  - Crack growth retardation remains in effect for a period related to the size of the plastic zone
    - The larger the plastic zone, the longer the crack growth retardation remains in effect

If the overload results in tensile residual stress, crack growth acceleration occurs.

![Figure 12: Crack growth retardation due to overloading](image)

Figure 12: Crack growth retardation due to overloading
Figure 13: Crack growth retardation of a structural steel

The Plastic zone size at a crack tip is given by the following equation:

$$r_y = \frac{1}{\beta \pi} \left( \frac{K}{\sigma_y} \right)^2$$

Where $\beta = 2$ for plane stress and $\beta = 6$ for plane stress.

Periodic overloads are not always beneficial. In low-cycle fatigue it may cause crack growth acceleration and need to be modelled applying the correct theories.

Compressive overload (sometimes referred to as underload) generally causes an acceleration in the crack growth rate because of tensile residual stress.

**Explanation of the mechanism of crack growth retardation**

The mechanism of the load sequence dependent crack growth retardation:

- Crack-tip blunting:
  - The crack tip blunts during overload $\Rightarrow$ stress concentration becomes less severe $\Rightarrow$ slower crack growth rate
  - Is not in line with practise where it was found that crack growth retardation comes in effect after crack has grown a portion through the plastic zone (*Delayed retardation*)

- Compressive residual stresses
  - Reduce the effective stress $\Rightarrow$ reduction in crack growth rate
  - Does not predict *delayed retardation*

- Crack closure models
  - Variations in opening stress $\Rightarrow$ stress intensity
  - This model does predict *delayed retardation*

**The next step is to develop models for crack closure**
8.12.1. Crack growth retardation prediction methods

In tests done on structural steels, it was found that when the baseline stress intensity range, $\Delta K$, is much greater than the threshold stress intensity, $\Delta K_{th}$ and plane stress conditions prevail, growth retardation is primarily due to plasticity-induced crack closure (Shin & Fleck, 1987, p. 392). For lower baseline stress intensity, $\Delta K_b$ close to the threshold stress intensity and plane strain conditions apply, an overload produces immediate crack arrest at the surface of the specimen, cut, not in the bulk of the specimen. Shin & Fleck postulated that this is due to combination of strain hardening and residual stress at the crack tip.

![Figure 14: Crack-tip plasticity](image)

The following approaches are typically used:

- **Statistical Methods**
  - Use the root mean square stress intensity factor
  - Only applicable to short spectra
  - Do not account for load sequence effects
  - Very restricted application
  - Does not predict crack growth retardation

- **Crack closure models**
  - Does predict crack growth retardation
  - Must estimate the opening stress for variable loading
  - Must be done cycle for cycle
  - Good correlation has been obtained

8.12.2. Further reading

(Carlson, Kardomateas, & Bates, 1991)
(Shin & Fleck, 1987)

8.13. Block loading

The application of block loading will be explained in class problems on fatigue. The principle behind block loading is to calculate damage per day, month, year, take-off-flight-landing, etc. event, and then calculated the number of times that the modelled event can be repeated. For the automotive industry one block is typically a mission profile driven length of route with known test route severity rating.

Block loading:

- Use blocks instead of cycle-for-cycle counting
- Considerable savings in time
- Limited to short spectra of loading
- Crack growth per block less than the plastic zone caused by the largest load cycle
- Damage is assumed to occur only when the crack is open
  - The crack opening stress must be determined
This means use of the change in positive stress intensity
  • Compressive residual stress beneficial

Dealing with service histories:
• Sometimes the service load history is unknown
• A representative load history or loading block may be determined from field tests
  – Analytical construction of service loads also done
• Fatigue life, or damage may then be calculated from the load blocks

Method: SAE Cumulative Damage Test program

8.14. Cumulative damage test program

• SAE Cumulative Damage Test Program
  – The component used in the study
  – Two steels were used:
    • ManTen
      – Yield strength = 80 ksi = 552 MPa
    • RQC-100
      – Yield strength = 120 ksi = 827 MPa
  – Different loadings were used

A series of tests were done to determine
  – Baseline material strain-life and crack growth data
  – Constant amplitude component load-life data
  – Variable amplitude component data

The following analysis techniques were used to predict lives
  – Rainflow counting was used to find ranges
  – Miner’s rule was used for damage summation
  – The life analysis was done using
  – Stress-life approach and the fatigue strength reduction factor $K_f$
  – Load-life curves
  – Local strain approach
    – Neuber analysis using $K_f$
    – Finite element analysis results
    – Assumption of elastic strain behaviour
    – Load-strain calibration curves using strain gauge measurements
  – Analysis were made ignoring and considering mean stresses
  – Techniques were also used to condense load histories
  – No analysis was made of crack propagation lives

Results of the program:
  – There was not a significant difference in the predictions made by any method that used a reasonable estimate of notch root stress-strain behaviour
Good predictions were made using the Neuber approach that tended to be slightly conservative.
There was not a large difference between predictions which included and excluded mean stresses.
Predictions made using the simple stress-life approach showed correlation which was as good as those predicted by more complicated techniques.
Another study showed that the following method predicted very good propagation lives:
- Use FEA to determine crack opening levels
- **Rainflow counting** + Linear Elastic Fracture Mechanics (LEFM)

### 8.15. Conclusion

- Miner’s linear damage rule provides reasonable life estimates
- Most effective cycle counting procedures relate damaging events to the stress-strain response of the material (Like Rainflow counting)
- Repeated block loading analysis techniques may be applied to save time
- Application of large overloads may cause crack growth retardation

### 9. CYCLE COUNTING WITH THE RAINFLOW METHOD

During this section Investmech will demonstrate to the student how the rainflow counting method works and how Matlab is applied in rainflow counting. For the purpose of this course the student is not expected to perform cycle counting, however, the student must be able to demonstrate the rainflow counting method.

Note, it is expected from the student to make notes of the rainflow counting method as explained by the lecturer in class. Notes will not be issued in class.

<table>
<thead>
<tr>
<th><strong>Presentation used in class:</strong></th>
<th>Cycle counting at Investmech</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filename:</td>
<td>Investmech - Structural Integrity (Cycle counting) R0.0</td>
</tr>
</tbody>
</table>

Use the remainder of this page and the following page to make notes during the lecture.
Leave open for notes.
10. STRESS LIFE ANALYSIS
This section introduces stress life analysis.

| Presentation used in class: | Investmech - Fatigue (Stress life analysis) R0.0 |

Discuss according to slides. The remainder of this section summarises problems done in class.

10.1. Slides not in notes
There are several slides presented in class that are not in the notes. Please download the slides to obtain digital copies of these slides. They are informative of nature, but, important and include:

– Total life curve
– Images of fracture surfaces

Notes from the lecture:
10.2. Notches
In general you were introduced to static failure criteria, of which maximum principal stress, von Mises and Tresca (maximum shear stress theory) are the most widely used theories. The stress-life approach cannot account for load sequence events, which can be modelled by the strain-life method. Local stress concentrations and residual stress are also accounted for in the strain-life method. Fracture mechanics is applied to modelled crack propagation after crack initiation.

10.2.1. Weld toes
Stress concentrations exist at weld toes. Accurate modelling of these stress concentration under low loads allows the use of the general fatigue curve. It is clear that cracks will initiate at the highest stressed points, which are mostly in the stress concentrations. Weld toes leave geometry that produce stress concentrations, the reason why cracks initiate in weld toes.

![Diagram of weld toes](image)

Source: (Taylor, Barrett, & Lucano, 2002)

Figure 15: Stress concentrations at weld toes result in crack initiation

10.3. Theoretic stress concentration factor
Stress concentrations (with theoretical stress concentration, $K_t$) due to geometrical or micro-structural discontinuities result in higher local stress, $\sigma_{sc}$, than the nominal stress away from the stress concentration, $\sigma$:

$$\sigma_{max} = K_t \sigma$$

$$K_t = \frac{\sigma_{sc}}{\sigma}$$ (25)

The ration between the maximum local stress and the nominal stress in a notched material is the theoretical stress concentration factor, $K_t$. $K_t$ is dependent on the geometry and the model of loading. Theoretical stress concentration factors for various geometries and modes of loading can be seen in Figure 16.
10.4. Notch fatigue factor

A fatigue notch factor is used to determine the relation between the endurance limit for a notched and un-notched specimen with the following equation:

\[ K_f = \frac{S_e^{(un-notched)}}{S_e^{(notched)}} \]  

(26)

Where:

- \( K_f \) Fatigue notch factor
The fatigue notch factor is also dependent on material type and can be calculated using the following equation:

\[ K_f = 1 + \frac{K_t - 1}{(1 + \frac{a}{r})} \]

\[ a = \frac{300}{f_u[ksi]} \times 10^{-3} \text{ in.} \]  \hspace{1cm} (27)

Where:

- \( a \) Factor depending on the material used [mm]
- \( r \) Notch radius [mm]

**Notch fatigue factor for \( K_t = 2 \)**

![Notch Fatigue Factor](image)

**Figure 17: Fatigue notch factor for \( K_t = 2 \)**

The fatigue notch factor can be used to adjust the S-N curve as can be seen in Figure 18.
Figure 18: S-N curve adjusted for a notched material

Ultimately, stress concentrations occurring from geometric discontinuities significantly lower than the fatigue life and endurance limit of a material.

An example of a finite element analysis done on a plate with a hole in the middle can be seen in Figure 19.

The applicable dimensions for the plate are:
- Hole diameter ($d$): 20 mm.
- Plate width ($w$): 50 mm.

This example demonstrates the stress concentration that exists in the regions next to the hole. The plate was loaded with a nominal stress of 100 MPa, and from Figure 19 it can be seen that the maximum stress is 334 MPa. The nominal stress in that area of the hole is $100 \times \frac{20}{50} = 167 \text{ MPa}$ and thus, the stress concentration factor is $\frac{334}{167} = 2$ according to the finite element analysis.

The same problem can be assessed with the stress concentration factors as can be seen in Figure 16. Thus the ratio is equal to $\frac{d}{w} = \frac{20}{50} = 0.4$ and from Figure 20 it can be seen that the stress concentration factor is roughly $K_f = 2.25$. Always make sure to calculate the stress in the area prescribed by the table. In some instances the nominal stress is calculated away from the stress concentrations, and in other cases the stress is calculated as average stress on remaining material at the stress concentration.
Figure 19: Finite element analysis of a plate with a hole in the middle

Include the questions and answers in the slides here.

Figure 20: Stress concentration factors for a plate with a hole in the middle
10.4.1. Notch fatigue factor at 1 000 cycles

The notch fatigue factor at 1 000 cycles can be calculated from the curve shown in Figure 21 and the following equation:

\[
\frac{K_f' - 1}{K_f - 1} = f(f_{ut})
\]  

(28)

![Figure 21: Ratio between notch fatigue factors at 1 000 and 1M cycles vs tensile strength](image)

10.5. Mean stress correction

In instances where dynamic loading with a specific non-zero mean stress occurs, such as with most practical systems, the Goodman equation is used to determine the completely reversed endurance limit of the material. The Goodman equation is given in the following equation:

\[
\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = 1
\]  

(29)

Where:

- \( S_e \)  Endurance limit [MPa]
- \( \sigma_a \)  Stress amplitude [MPa]
- \( \sigma_m \)  Mean stress [MPa]
- \( S_{ut} \)  Ultimate tensile strength [MPa]

A normalized amplitude-mean diagram which yields a straight line approximation called a Goodman or Haigh diagram can be used to assess life under non-zero mean stress conditions. An example of a Haigh diagram showing the regions of finite and infinite life for a specific alternating and mean stress can be seen in Figure 22.
Figure 22: Haigh diagram sketch showing the regions for finite and infinite life
A typical Haigh diagram for various mean and alternating stress conditions is shown in Figure 23.

![Haigh diagram sketch showing the regions for finite and infinite life](image)

Figure 23: Haigh diagram for AISI 4340 steel (Bannantine et al., 1990:7)

### 10.6. Modifying factors

The objective of modifying factors is to calculate the endurance limit and the fatigue strength at 1000 cycles under various conditions.

\[
S_e = S'_e C_{size} C_{load} C_{surf} C_T C_{rel} \\
S_{10^3} = S'_{10^3} C_{load} C_T C_{rel}
\]

The general trend of modifying factors is to have less effect at short lives. However, some, like reliability, need to be included. The modifying factors that is expected to affect the fatigue life at low cycle end are \(C_{load}, C_T\) and \(C_{rel}\).

#### 10.6.1. Size factor on different shaft sizes

The size factor as function of shaft diameter is as follows:

\[
C_{size} = \begin{cases} 
1.0, & \text{if } d \leq 8 \text{ mm} \\ 
1.189d^{-0.97}, & \text{if } 8 \text{ mm} < d \leq 250 \text{ mm} 
\end{cases}
\]

Table 2 summarises a few values.
Table 2: Shaft diameter dependent size factor

<table>
<thead>
<tr>
<th>d [mm]</th>
<th>C_size</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.95</td>
</tr>
<tr>
<td>30</td>
<td>0.85</td>
</tr>
<tr>
<td>50</td>
<td>0.81</td>
</tr>
<tr>
<td>70</td>
<td>0.79</td>
</tr>
<tr>
<td>90</td>
<td>0.77</td>
</tr>
<tr>
<td>110</td>
<td>0.75</td>
</tr>
<tr>
<td>130</td>
<td>0.74</td>
</tr>
<tr>
<td>150</td>
<td>0.73</td>
</tr>
<tr>
<td>170</td>
<td>0.72</td>
</tr>
<tr>
<td>190</td>
<td>0.71</td>
</tr>
<tr>
<td>210</td>
<td>0.71</td>
</tr>
<tr>
<td>230</td>
<td>0.70</td>
</tr>
<tr>
<td>250</td>
<td>0.70</td>
</tr>
</tbody>
</table>

10.6.2. Size factor on thickness

The size factor, $k_b$, is given by the following for bending and torsion (Budynas & Nisbett, 2012, p. 280):

$$k_b = 1.51d^{-0.157} \quad (32)$$

Where:

$d$ Section thickness, taken as the trunnion wall thickness [m]

10.6.3. Loading effects

A conservative relationship due to the volume subject to high stress is as follows:

$$S_{e,axial} = 0.70S_{e,bending}$$

$$C_{load} = 0.7 \text{ if } S - N \text{ curve is from bending tests} \quad (33)$$

If the S-N curve was constructed from a bending test, the load factor for a specimen subjected to axial loading will be approximately 0.7. However, if the S-N curve used was constructed from axial tests, the load factor for a specimen loaded in bending will be $\frac{1}{0.7} = 1.43$.

For torsion (see the slides for the derivation):

$$\tau_e(torsion) = 0.577S_{e,bending} \quad (34)$$

10.6.4. Surface finish and corrosion

A corrosive environment attacks the surface of a material and produces an oxide film. Cyclic loading causes localized cracking of the layer, exposing the fresh material to the corrosive environment. This causes localized pitting where stress concentration occurs. Corrosion reduces the fatigue life of a material and eliminates the endurance limit. Thus, if an unprotected material operates in a corrosive environment, the material’s endurance limit will fall away and cannot be taken into account.

Surface finish modification factors which can be used to modify the endurance limit of steel is given in Figure 24. Modification factors for steel corroded in water, as well as in salt water, are also given in Figure 24 for steels with different ultimate tensile strengths.

Surface treatment can have a significant effect on fatigue life, because the crack initiates at the free surface. In plating, thermal and mechanical treatment (welding, milling, pressing, etc.), the effect on fatigue life is primarily due to residual stress. Should a residual stress result, pre-stressing or pre-setting (initial overloading of the component, which is only favourable for fatigue loads in the direction of the overload) should be done to produce compressive residual stress at the free surface.
According to BS 7608 (1993), for unprotected joints exposed to water (specifically sea water) the basic S-N curves should be reduced by a factor of 2 on life for all joint classes. For high strength steels, with yield strengths of higher than 400 MPa, this factor may not be adequate and the use of such materials under corrosion fatigue conditions should be approached with caution.

10.6.4.1. Plating
Chrome and Nickel plating of steels can cause up to 60% reduction in endurance limits. High tensile stresses are generated by plating process. To alleviate residual stress problem:

- Nitride part before plating
- Shot peen part before or after plating (Best to peen after plating)
- Bake or anneal the part after plating

Corrosion resistance offered by plating can more than offset the reduction in fatigue strength seen in non-corrosive environment. Plating with cadmium and zinc appear to have no effect on fatigue strength. Electroplating can cause hydrogen embrittlement.

10.6.5. Temperature
- Diffusion processes such as carburizing and nitriding beneficial for fatigue strength
  - Produces higher strength material on surface
  - Causes volumetric changes that produce residual compressive stresses
- Flame and induction hardening
  - Cause phase transformation which in turn cause volumetric expansion

---

**Figure 24: Surface finish modification factors for steel (ASM International, 2008:16)**
– If localized to surface – compressive residual stresses result that is beneficial for fatigue strength
  • Hot rolling and forging
    – Cause surface decarburization
    – Loss of carbon atoms from surface causes lower strength and may produce residual stresses
    – Both factors are detrimental to fatigue strength
  • Manufacturing processes such as grinding, welding, flame cutting etc.
    – Can set up detrimental residual tensile stresses
    – Shot peening effective to undo damage caused by these processes

• Endurance limits of steels increase at low temperatures (watch out for brittleness)
• Endurance limit for steels disappears at high temperatures due to mobilization of dislocations
• For $T > T_{\text{mol}}/2$ creep becomes important
  – Stress-life approach no longer applicable.
• Annealing happens at high temperatures that may remove beneficial residual compressive stresses

According to Roymech:

$$C_T = \begin{cases} 1.0 & \text{for } T \leq 450^\circ \text{C} \\ 1 - 5.8^{-3}(T - 450), & \text{for } 450 < T \leq 550^\circ \text{C} \end{cases}$$ (35)

10.6.6. Reliability
The probability of survival, or failure, is statistically driven. BS 7608 provides means to shift various S-N fatigue curves for client specified probability of survival.

<table>
<thead>
<tr>
<th>Reliability $1 - p_f$</th>
<th>$C_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>0.9</td>
<td>0.897</td>
</tr>
<tr>
<td>0.95</td>
<td>0.868</td>
</tr>
<tr>
<td>0.99</td>
<td>0.814</td>
</tr>
<tr>
<td>0.999</td>
<td>0.753</td>
</tr>
<tr>
<td>0.9999</td>
<td>0.702</td>
</tr>
<tr>
<td>0.99999</td>
<td>0.659</td>
</tr>
<tr>
<td>0.999999</td>
<td>0.620</td>
</tr>
</tbody>
</table>

10.6.7. Mechanical modifying factors
• Cold work processes – rolling & shot peening
  – Produce compressive residual stresses
    • Gives the greatest improvement in fatigue life
  – Work-hardens the material
  – Rolling cause deep stress layer (bolts, etc)
  – Shot peening gives (compressive stress = $0.5S_y$) layer of ~1 mm
  – Shot peening:
    • leaves dimpled surface: hone or polish part after shot peening
• Undo deleterious effects caused by chrome and nickel plating, decarburization, corrosion, grinding, etc.
• Steels with \( F_y \leq 550\) MPa seldom cold rolled or shot peened (Easy to introduce plastic strains that wipe out residual stresses)
• Surfaces can be overpeened! Subsurface failures may occur!
  
- Loading frequency
  - Similar data at various frequency in non-corrosive environment
  - Corrosion-fatigue are greatly influenced by loading frequency

10.7. Derivation of S-N curve for \( S_{1000} \) and \( S_e \) modified by the fatigue notch factors

From the theory presented in class, the fatigue notch factors modify the S-N curve as follows:

\[
S'_{1000} = \frac{S_{1000}}{K'_f} \\
S'_e = \frac{S_e}{K_f}
\]  

(36)

The S-N curve is modelled by equation: \( S^{m} = C N^b \). Using this relation, the coefficient and exponent in the notch modified S-N curve equation can be solved as follows:

\[
\left( \frac{S'_{1000}}{S'_e} \right)^m = 10^3
\]

\[
m = 3 \log \frac{S'_{1000}}{S'_e}
\]  

(37)

10.8. Endurance calculation example

Problem statement:

300WA structural steel has the following material properties:

- \( E = 206 \) GPa
- \( S_y = 300\) MPa
- \( S_{ut} = 450\) MPa

Assume a notch fatigue factor of \( k_f = 1.7 \). What is the endurance limit? How many cycles to failure at:

1. \( S_a = 200\) MPa
2. \( S_a = 300\) MPa

Assumptions:

Investmech assumed that the fatigue notch factor at 1000 cycles needs to be estimated from the material properties supplied. This information was not provided in the problem statement, but, will be inferred from the class notes.

The S-N curve can be described as: \( S = C N^b \).

Solution:

Figure 25 shows the relationship of the following ratio to the material ultimate tensile strength of 490 MPa (70 ksi) as approximately 0.15, from which the fatigue notch factor at 1,000 cycles is \( K'_f = 1.1 \):

\[
\frac{K'_f - 1}{K_f - 1} = 0.15 \\
K'_f = 0.15K_f + 0.85 \\
= 0.15 \times 1.7 + 0.85 \\
= 1.1
\]  

(38)
From this, the end points (at 1,000 and 1,000,000 cycles) of the S-N curve are as follows:

\[
S'_{1000} = \frac{S_{1000}}{K_f'} = \frac{0.9S_{ut}}{1.1} = C10^{3b}
\]

\[
S'_{e'} = \frac{S_{e'}}{K_f'} = \frac{0.5S_{ut}}{1.7} = C10^{6b}
\]

(39)

Dividing the two equations gives:

\[
\frac{S'_{e'}}{S'_{1000}} = 10^{3b}
\]

(40)

The coefficient \(C\) is then:

\[
C = \frac{S_{1000}}{10^{3b}}
\]

\[
C = \frac{S'_{e'}}{10^{6b}}
\]

(41)

The values are as summarised in the Excel sheet below from which the following may be concluded:

1. The endurance limit for the notched specimen is 144 MPa.
2. The number of cycles to failure, which is also called the endurance, for 50% probability of survival at stress amplitude 200 MPa is 108,358 cycles.
3. The number of cycles to failure, also called the endurance, for 50% probability of survival at stress amplitude 300 MPa is 6,929 cycles.

**Table 3: Example fatigue calculation**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Sy</td>
<td>300 MPa</td>
</tr>
<tr>
<td>Sut</td>
<td>490 MPa</td>
</tr>
<tr>
<td>Kf</td>
<td>1.7</td>
</tr>
<tr>
<td>(K_f' - 1)</td>
<td>0.15</td>
</tr>
<tr>
<td>(K_f - 1)</td>
<td>1.105</td>
</tr>
</tbody>
</table>

From the graph \(K_{f'} 1.105\)

Calculations

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Endurance limit for the notched specimen</td>
<td>144.1176 MPa</td>
</tr>
<tr>
<td>b</td>
<td>-0.14745</td>
</tr>
<tr>
<td>C=</td>
<td>1105.186</td>
</tr>
<tr>
<td>Check:</td>
<td>1105.186</td>
</tr>
</tbody>
</table>

Endurance for stress limits

<table>
<thead>
<tr>
<th>Stress amplitude</th>
<th>Endurance</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>108,358</td>
</tr>
<tr>
<td>300</td>
<td>6,929</td>
</tr>
</tbody>
</table>
10.9. Fatigue life example with mean stress

Problem statement:
Component undergoes cyclic stress as follows:

<table>
<thead>
<tr>
<th>Stress amplitude [MPa]</th>
<th>Stress mean [MPa]</th>
<th>Number of cycles/block</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>200</td>
<td>10000</td>
</tr>
<tr>
<td>500</td>
<td>0</td>
<td>5000</td>
</tr>
<tr>
<td>600</td>
<td>100</td>
<td>20000</td>
</tr>
<tr>
<td>700</td>
<td>-100</td>
<td>2000</td>
</tr>
</tbody>
</table>

The material is steel with $S_{ut} = 1,050\text{MPa}$ and $S_e = 420\text{MPa}$. The fully reversed stress at $S_{1000} = 770 \text{ MPa}$.

How many blocks of loading can be loaded on the component until fatigue crack initiation? That is, what is the fatigue life of the component?

Solution:
The solution is presented in the table below. Goodman correction was done. In general, the steps are as follows:

1. Calculate the notch fatigue factor at 1,000 and 1,000,000 cycles.
2. Solve the S-N curve coefficient and exponent.
3. Find the signal mean and amplitude.
4. Do Goodman correction to find the completely reversed stress amplitude.
5. Calculate the endurance for each completely reversed stress amplitude.
6. Calculate damage for each completely reversed stress amplitude.
7. Sum the to find total damage.
8. Failure is when total damage is equal to 1. Calculate the life proportionally.
### Calculations

<table>
<thead>
<tr>
<th>Sy</th>
<th>?? MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sut</td>
<td>1050 MPa</td>
</tr>
<tr>
<td>Kf</td>
<td>1</td>
</tr>
</tbody>
</table>

\[
\frac{K_f' - 1}{K_f - 1} = 0.15 \quad \text{From the graph} \\
K_f' = 1
\]

\[
\begin{align*}
\frac{0.55_{\text{ MPa}}}{1.7} & = 10^{3.0} \\
\frac{0.95_{\text{ MPa}}}{1.1} & = 10^{2.0}
\end{align*}
\]

**Calculations**

| S1000' | 945 |
| Size factor | 1 |
| Surface factor | 1 |
| Temperature factor | 1 |
| Reliability factor | 1 |
| Endurance limit for the notched specimen | 525.0 MPa |
| b | -0.085 |
| C= | 1701.0 |

**Check:** 1701

<table>
<thead>
<tr>
<th>Stress amplitude</th>
<th>Mean stress</th>
<th>ni</th>
<th>Compl. Rev. Stress Sa</th>
<th>Ni</th>
<th>Di</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>200</td>
<td>10000</td>
<td>494</td>
<td>inf</td>
<td>0.00</td>
</tr>
<tr>
<td>500</td>
<td>0</td>
<td>5000</td>
<td>500</td>
<td>inf</td>
<td>0.00</td>
</tr>
<tr>
<td>600</td>
<td>100</td>
<td>20000</td>
<td>663</td>
<td>64218</td>
<td>0.31</td>
</tr>
<tr>
<td>700</td>
<td>-100</td>
<td>2000</td>
<td>639</td>
<td>99086</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Total Damage 0.33
Duration 1 Blocks
Life 3.02 Blocks

### 10.10. Conclusion

- Endurance limit only exist in plain carbon and low-alloy steels
- Following factors will reduce the endurance limit:
  - Tensile mean stress, large section size, rough surface finish, chrome and nickel plating (except in corrosive environment), decarburization due to forging and hot rolling, severe grinding
- Following factors tend to increase the endurance limit:
  - Nitriding, flame and induction hardening, carburization, shot peening, cold rolling
  - Chrome and nickel plating for materials in a corrosive environment
11. STRAIN LIFE ANALYSIS

This section introduces strain life analysis. This is for specialist application only and no notes will be issued in class. A presentation will be done just to introduce the concept to students. No assessment in assignments or examination will be done on this topic. Students may download the documents listed below for information purposes.

<table>
<thead>
<tr>
<th>Presentation used in class:</th>
<th>Investmech - Fatigue (Strain life analysis) R0.0</th>
</tr>
</thead>
</table>
| Download link for the Excel sheet for strain life sequence effects: | Document name: Strain life example - Sequence effect.xls  
Link: [http://www.investmech.com/fatigue.html](http://www.investmech.com/fatigue.html) |

12. References


