Department of Mechanical and Aeronautical Engineering

University of Pretoria

Open Book Exam

(Allowed: Textbook, Handout notes, Class notes, downloads from the website)

Not allowed: Laptop computers, communication devices

MSV 780 Fatigue

Time: 3 Hours **Full Marks:** 180

Examiner: Dr. Michiel Heyns **Date:** 14 June 2019

External Examiner: Prof Madeleine du Toit, University of Wollongong, Australia

Instructions:

- Clarifications or other information may be written on the blackboard -- check it before handing in your paper.
- Logic and calculations leading to all answers must be given, and any points or lines used from graphs must be marked and labelled where applicable.
- For discussion or explanation type questions, answers need to be in complete sentences and in reasonably good English, and the logic must be clear.
- Numerical answers should have neither insufficient nor excessive numbers of significant figures.
- Correct units are required for full credit.

Good luck!

1 STRESS-LIFE PROBLEM [60 Marks]

A steel quenched and tempered bar is subject to the rotating bending stress spectrum shown in Table 1. The stress spectrum was determined from measured strains, converted to stress and then Rainflow-counted.

The yield strength of the material is $\sigma_o = 683\ MPa$ and ultimate tensile strength $\sigma_u = 758\ MPa$. The roller is subject to bending stress. Assume that a fatigue curve for 50% probability of survival of a mirror polished unnotched specimen has the following endurances and completely reversed bending stress amplitudes $(N;\ \sigma_{ar})$:

- $(1\,000; 0.9\sigma_u)$
- $(1\,000\,000; \sigma_{erb})$, where the completely reversed endurance limit in bending is: $\sigma_{erb} = 0.5\sigma_u$

The S-N curve has the following relationship

$$\sigma_{ar,1}N_1^b = \sigma_{ar,2}N_2^b = C$$

The roller has a notch with radius 10 mm with theoretical stress concentration factor $k_t = 1.2$. The roller diameter is 200 mm.

The operating parameters are:

- Temperature 300 °C
- Surface finish: Hot rolled
- Non-corrosive environment and the part is surface protected

Construct your S-N curve then please answer the following:

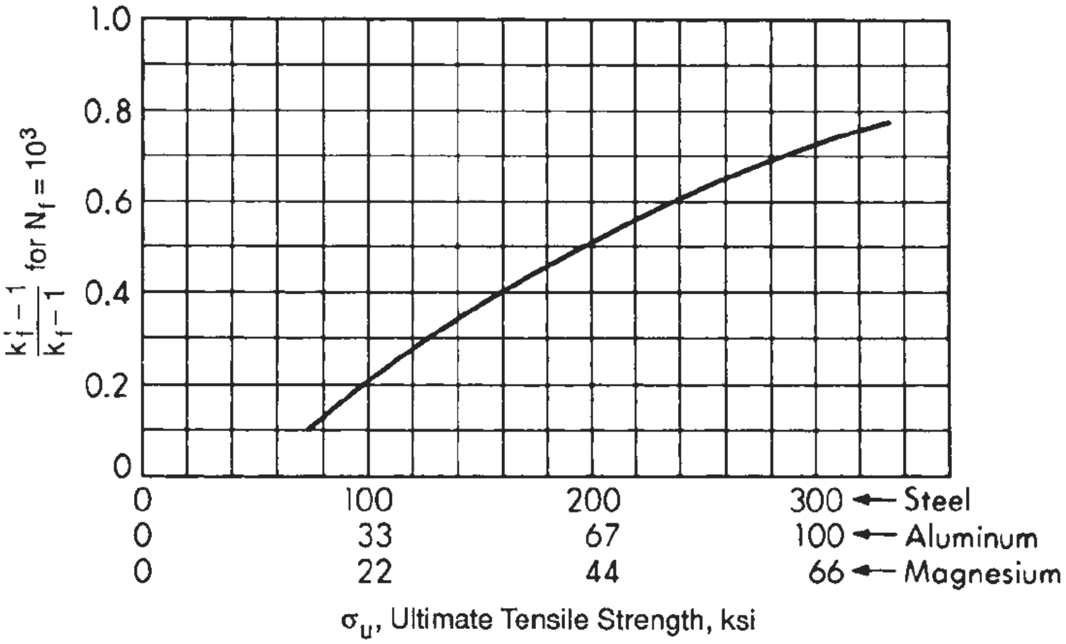
1. Use the modified Goodman mean stress compensation method and calculate the following for a 99% probability of survival **[Marks =55]:**
   a. Repetitions of the stress spectrum that can be applied before crack initiation **[95% x mark]**.
   b. The fatigue life in years **[5% x mark].**
2. Comment on the use of the modified Goodman mean stress compensation in this case **[5 Marks].**

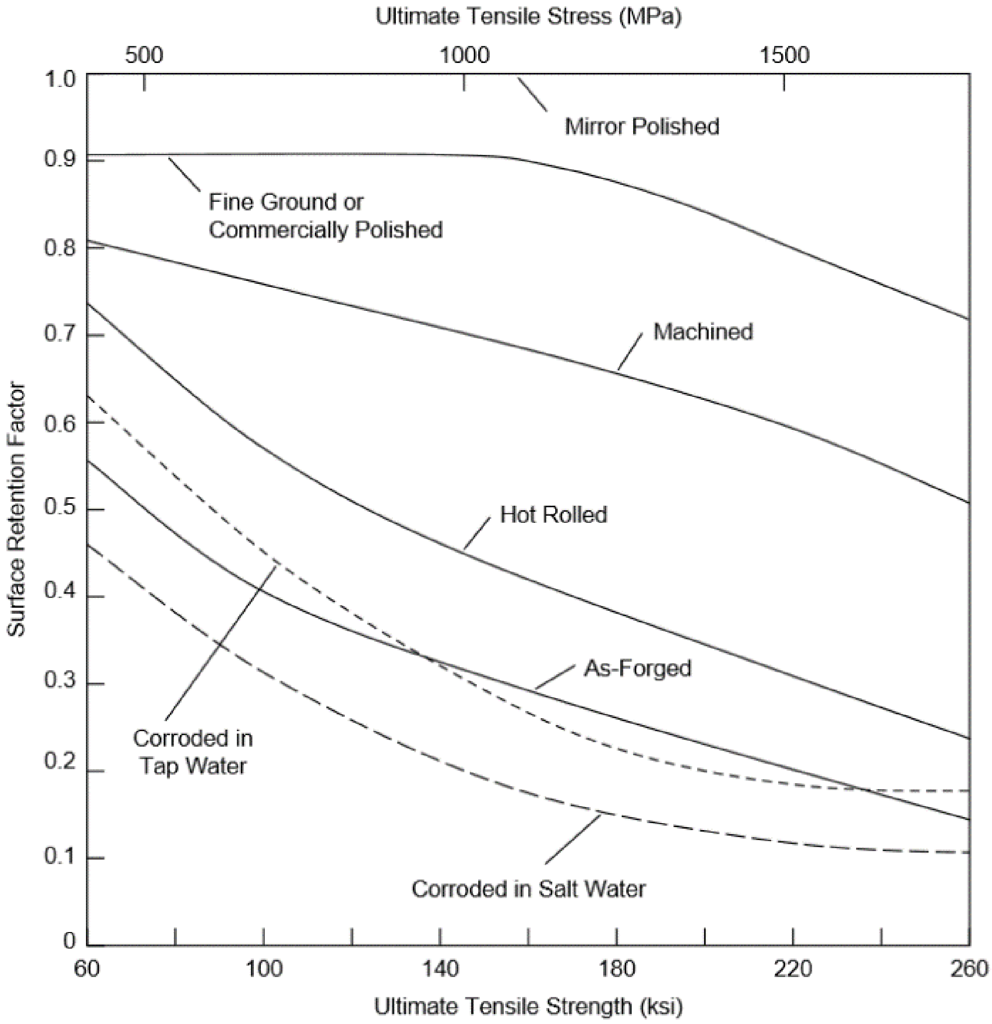
Please tear out the next page, put your student number on and submit with your answering sheet.

Table 1: Rotating bending stress spectrum over a period of 1 year

σ_a [MPa]	σ_m [MPa]	n [cycles]
100	100	10 000
150	0	20 000
100	-50	5 000

1.1 Hand in with your paper: Student No:__________________________________.





1.2 Solution

Question 1

Determine the values on the S-N curve

Because there is a notch involved, the equation for the fatigue strength at 1 000 cycles, $\sigma'_{arb,10^3}$, and the endurance limit at 1 000 000 cycles, σ'_{erb}, is:

$$\sigma'_{erb} = \frac{0.5\sigma_u}{K_f}$$

$$\sigma'_{arb,10^3} = \frac{0.9\sigma_u}{K'_f}$$

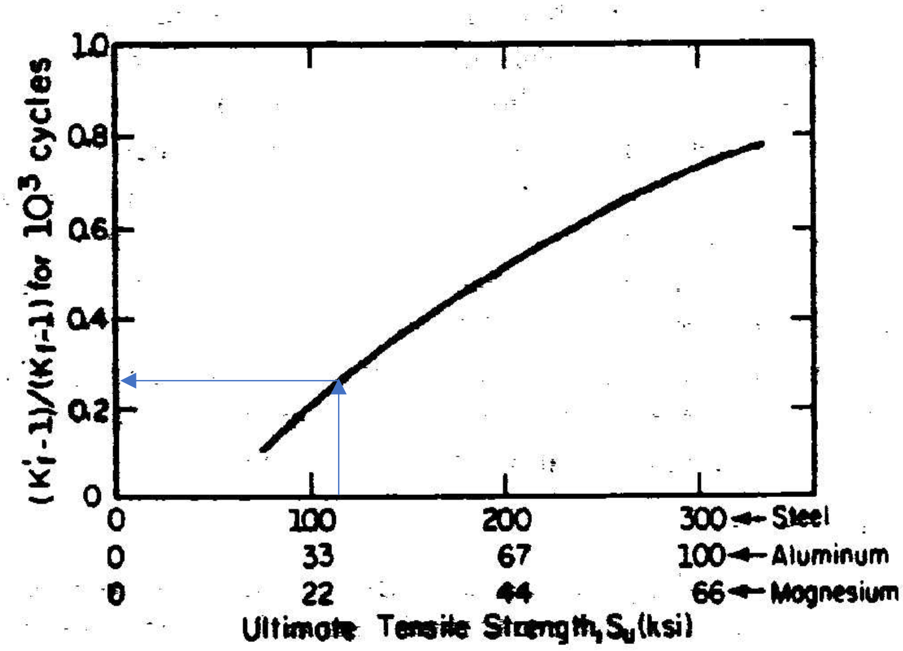
The fatigue notch factors are given by:

$$\begin{aligned} a &= \left[\frac{300}{\sigma_u[ksi]}\right]^{1.8} \times 10^{-3} \times 25.4 \\ &= \left[\frac{300}{\left(\frac{758}{6.89}\right)}\right]^{1.8} \times 10^{-3} \times 25.4 \\ &= 0.155\, mm \end{aligned}$$

The fatigue notch factor at the endurance limit is:

$$\begin{aligned} K_f &= 1 + \frac{K_t - 1}{\left(1 + \frac{a}{r}\right)} \\ &= 1 + \frac{1.2 - 1}{1 + \frac{0.155}{10}} \\ &= 1.20 \end{aligned}$$

The fatigue notch sensitivity factor, q, is approximately 0.25 from the figure below:



Equation for the S-N curve

The fatigue notch factor at 1 000 cycles is then:

$$\begin{aligned}\frac{K_f'-1}{K_f-1} &= f(\sigma_u) = q\\ K_f' &= q\left(K_f-1\right)+1\\ &= 0.25(1.20-1)+1\\ &= 1.05\end{aligned}$$

Allocate 10% of the mark for the fatigue notch factor calculations

The modified S-N curve has the following points:

$$\begin{aligned}\sigma_{er} &= \sigma_{\mathrm{erb}}' C_{size} C_{load} C_{surf} C_T C_{rel}\\ \sigma_{erb}' &= \frac{0.5\sigma_u}{K_f}\\ \sigma_{ar,10^3} &= \sigma_{arb,10^3}' C_{load} C_T C_{rel}\\ \sigma_{arb,10^3}' &= \frac{0.9\sigma_u}{K_f'}\end{aligned}$$

Modifying factor: size, C_{size}

For the shaft with diameter 200 mm, the size factor is:

$$\begin{aligned}C_{size} &= \begin{cases} 1.0, if\ d \le 8\ mm \\ 1.189d^{-0.097}, if\ 8\ mm < d \le 250\ mm\end{cases}\\ &= 1.189(200)^{-0.097}\\ &= 0.71\end{aligned}$$

Allocate 5% of the mark

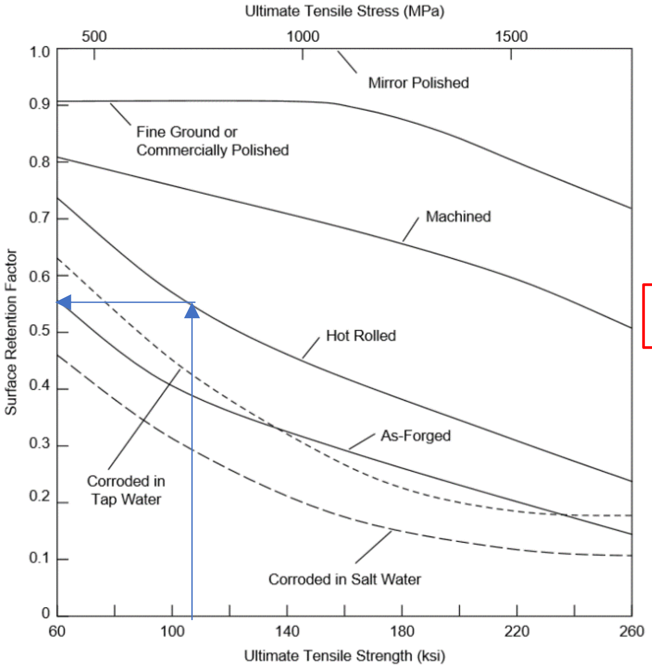
Modifying factor: Load, C_{load}

In this case the problem asks for an analysis of a roller subject to bending. The S-N curve used, is also for a specimen in bending. Therefore, $C_{load} = 1.0$.

Allocate 5% of the mark who motivated this

Modifying factor: surface, C_{surf}

The S-N curve was for a mirror polished specimen. The fatigue analysis is to be done on a hot rolled part. Therefore, the reduction factor for surface finish is $C_{surf} = 0.55$.



Allocate 5% of the mark

Modifying factor: reliability, C_{rel}

The fatigue curve to be constructed uses as reference the strengths at 1 000 and 1 000 000 cycles for 50% probability of survival in log(N). To have a reliability of 99%, the modification factor is: $C_{rel} = 0.814$.

Reliability $1 - p_f$	C_r
0.5	1
0.9	0.897
0.95	0.868
0.99	0.814
0.999	0.753
0.9999	0.702
0.99999	0.659
0.999999	0.620

Allocate 5% of the mark

Modification factor: temperature, C_T

A modification factor for temperature 300 °C is given below, from which:

$$\begin{aligned} C_T &= \begin{cases} 1.0 \; for \; T \leq 450 \text{ °C} \\ 1 - 5.8^{-3}(T - 450), \; for \; 450 < T \leq 550 \text{ °C} \end{cases} \\ &= 1.00 \end{aligned}$$

Allocate 5% of the mark

Two points on the S-N curve

The two points to use on the modified S-N curve are:

- At 1 000 cycles:

$$\begin{aligned} \sigma'_{arb,10^3} &= \frac{0.9\sigma_u}{K'_f} \\ \sigma_{ar,10^3} &= \sigma'_{arb,10^3} C_{load} C_T C_{rel} \\ &= \frac{0.9\sigma_u}{K'_f} C_{load} C_T C_{rel} \\ &= \frac{0.9 \times 758}{1.05} \times 1 \times 1 \times 0.814 \\ &= 529 \text{ MPa} \end{aligned}$$

Allocate 5% of the mark for the student who understands this process and who obtained a fatigue strength within 10% of the value

- At 1 000 000 cycles, the endurance limit for completely reversed loading is:

$$\begin{aligned} \sigma'_{erb} &= \frac{0.5\sigma_u}{K_f} \\ \sigma_{er} &= \sigma'_{\text{erb}} C_{size} C_{load} C_{surf} C_T C_{rel} \\ &= \frac{0.5\sigma_u}{K_f} C_{size} C_{load} C_{surf} C_T C_{rel} \\ &= \frac{0.5 \times 758}{1.2} \times 0.71 \times 1 \times 0.55 \times 1 \times 0.814 \\ &= 100 \text{ MPa} \end{aligned}$$

Allocate 5% of the mark for the student who understands this process and who obtained a fatigue strength within 10% of the value

With the fatigue strengths known at 1 000 and 1 000 000 cycles, determine S-N curve parameters

The equation for the S-N curve is assumed to be:

$$\sigma_{ar,1} N_1^b = \sigma_{ar,2} N_2^b$$

Therefore:

$$b \log\frac{N_1}{N_2} = \log\frac{\sigma_{ar,2}}{\sigma_{ar,1}}$$

$$\begin{aligned} b &= \frac{\log\frac{\sigma_{ar,2}}{\sigma_{ar,1}}}{\log\frac{N_1}{N_2}} \\ &= \frac{\log\frac{\sigma_{er}}{\sigma_{ar,10^3}}}{\log\frac{N_{10^3}}{N_e}} \\ &= \frac{\log\frac{100}{529}}{\log\frac{1\,000}{1\,000\,000}} \\ &= 0.241 \end{aligned}$$

Allocate 5% of the mark for the student who understands this process and who obtained a fatigue strength within 10% of the value

The endurance at any completely reversed stress amplitude is then:

$$N_R = \begin{cases} \left(\frac{\sigma_{er}}{\sigma_{ar}}\right)^{\frac{1}{b}} N_e & \sigma_{ar} \geq \sigma_{er} \\ \infty & \sigma_{ar} < \sigma_{er} \end{cases}$$

Mean stress compensation

The problem statement mentioned that the modified Goodman method should be used for mean stress compensation. From the modified Goodman mean stress compensation, the equivalent completely reversed stress for positive maximum stress is:

$$\sigma_{ar} = \frac{\sigma_a}{1 - \frac{\sigma_m}{\sigma_u}}$$

Apply to the stress spectrum

σ_a [MPa]	σ_m [MPa]	n [cycles]
100	100	10 000
150	0	20 000
100	-50	5 000

For amplitude 100 MPa, mean 50 MPa and 10 000 cycles, the damage per repetition is:

1. The mean stress corrected completely reversed amplitude:

$$\begin{aligned} \sigma_{ar,1} &= \frac{\sigma_a}{1 - \frac{\sigma_m}{\sigma_u}} \\ &= 115\ MPa \end{aligned}$$

2. This completely reversed stress exceeds the endurance limit, σ_{er}, and the endurance is:

$$\begin{aligned} N_R &= \begin{cases} \left(\frac{\sigma_{er}}{\sigma_{ar}}\right)^{\frac{1}{b}} N_e & \sigma_{ar} \geq \sigma_{er} \\ \infty & \sigma_{ar} < \sigma_{er} \end{cases} \\ &= \left(\frac{100}{115}\right)^{\frac{1}{0.241}} \times 1 \times 10^6 \\ &= 559\,940\ cycles \end{aligned}$$

3. The damage is then:

$$D_1 = \frac{10\,000}{559\,940} = 0.0179$$

5% for correct mean stress compensation

5% for endurance within 10%

5% for correct damage

For amplitude 150 MPa, mean 0 MPa, and, 20 000 cycles the damage is:

1. The mean stress corrected completely reversed amplitude:

$$\sigma_{ar,2} = 150\ MPa$$

2. This completely reversed stress exceeds the endurance limit, σ_{er}, and the endurance is:

5% for correct mean stress compensation

5% for endurance within 10%

$$N_R = \begin{cases} \left(\frac{\sigma_{er}}{\sigma_{ar}}\right)^{\frac{1}{b}} N_e & \sigma_{ar} \geq \sigma_{er} \\ \infty & \sigma_{ar} < \sigma_{er} \end{cases}$$

$$= \left(\frac{100}{150}\right)^{\frac{1}{0.241}} \times 1 \times 10^6$$

$$= 185\,920\ cycles$$

3. The damage is then:

$$D_2 = \frac{20\,000}{185\,920} = 0.108$$

For amplitude 100 MPa, mean -50 MPa, and 5 000 cycles, the damage is:

1. The mean stress corrected completely reversed amplitude:

$$\sigma_{ar,3} = \frac{100}{1 - \frac{-50}{758}}$$

$$= 94\ MPa$$

2. The completely reversed stress range is less than the endurance limit and the endurance is infinite: $N_R = \infty$.
3. The damage is then zero:

$$D_3 = \frac{5\,000}{\infty} = 0$$

5% for correct damage

The total damage per repetition is then:

$$D = \sum D_i$$

$$= 0.108 + 0.018 + 0$$

$$= 0.126$$

The number of repetitions to failure is then:

$$B_f = \frac{1}{D}$$

$$= \frac{1}{0.126}$$

$$= 7.9$$

10% for answer within 10%

The period of one repetition was 1 year, therefore, the fatigue life for a 95% probability of survival is:

$$Life = B_f \times 1\ year$$

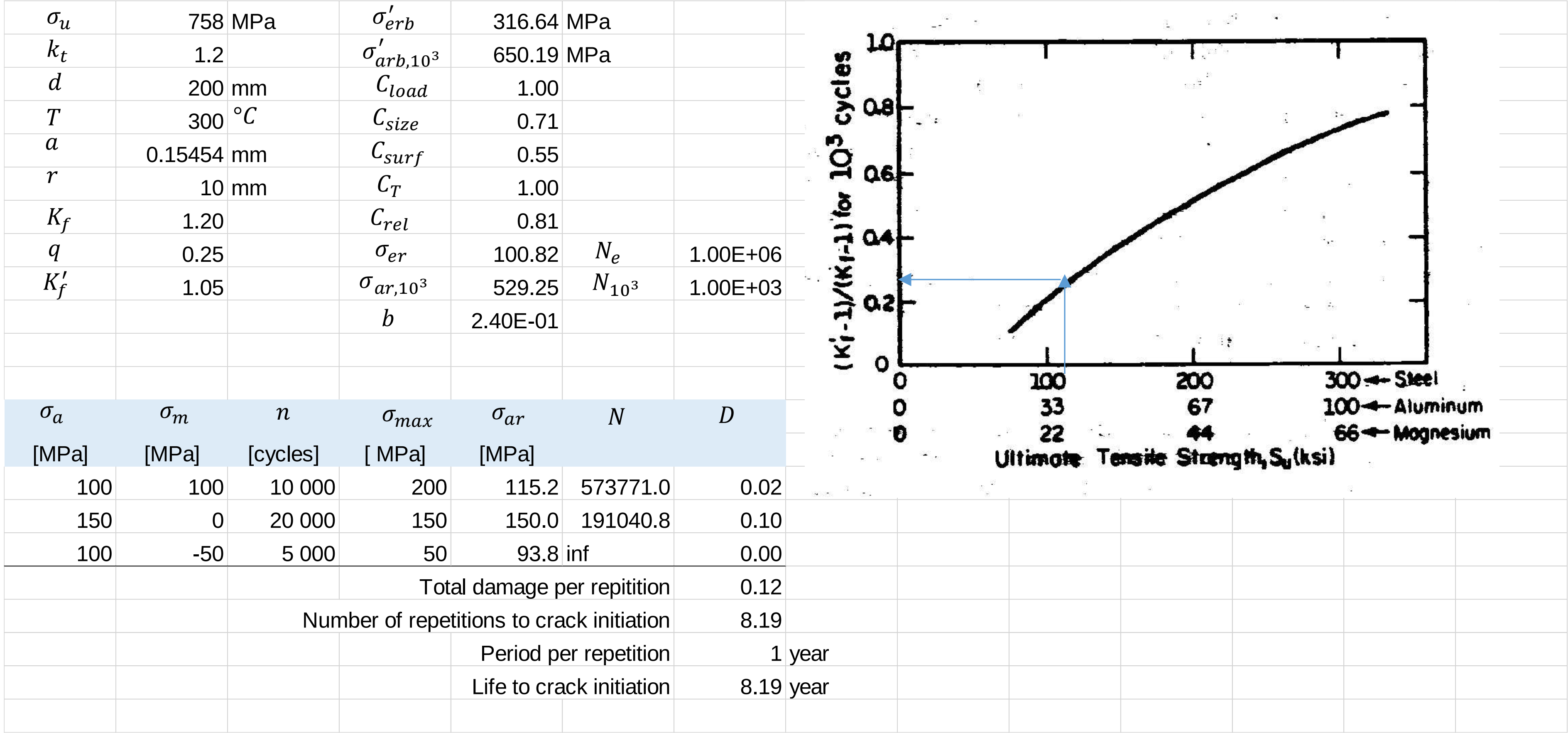
$$= 7.9\ years$$

5% for answer within 10%

For a candidate who followed the process above, or used another equation to fit the S-N curve:

1. **100% if all aspects have been considered and the answer is within 10%**
2. **Subtract for aspects not included in the assessment e.g.:**
    a. **mean stress compensation;**
    b. **temperature;**
    c. **load;**
    d. **surface finish, and,**
    e. **size.**

Check answer with Excel spreadsheet



σ_u	758	MPa	σ'_{erb}	316.64	MPa		
k_t	1.2		$\sigma'_{arb,10^3}$	650.19	MPa		
d	200	mm	C_{load}	1.00			
T	300	$°C$	C_{size}	0.71			
a	0.15454	mm	C_{surf}	0.55			
r	10	mm	C_T	1.00			
K_f	1.20		C_{rel}	0.81			
q	0.25		σ_{er}	100.82	N_e	1.00E+06	
K'_f	1.05		$\sigma_{ar,10^3}$	529.25	N_{10^3}	1.00E+03	
			b	2.40E-01			

σ_a [MPa]	σ_m [MPa]	n [cycles]	σ_{max} [MPa]	σ_{ar} [MPa]	N	D
100	100	10 000	200	115.2	573771.0	0.02
150	0	20 000	150	150.0	191040.8	0.10
100	-50	5 000	50	93.8	inf	0.00

Total damage per repitition	0.12	
Number of repetitions to crack initiation	8.19	
Period per repetition	1	year
Life to crack initiation	8.19	year

Question 2

The material used in this problem is steel.

- Modified Goodman was demonstrated in class to give a good correlation for steel, and, makes provision for compressive stresses. It is a conservative method and can be used where there are marginal uncertainties present in material fatigue properties and/or loading.
- Morrow relationship with σ_f', the ordinate at one reversal (half-cycle), fits data well for steels. However, in this case the modified fracture strength is not available.
- SWT was found to be a good choice for Aluminium.
    - Because the SWT mean stress compensation does not rely on any material constant, it is an attractive method to use that give reasonable results for Aluminium alloys.
    - SWT gives reasonable results for steel.

Allocate marks according to the candidate's reasoning of the above.

2 STRAIN LIFE [60 Marks – one mark/minute]

Figure 1 shows strain vs life curves for RQC-100 steel. For each of the several tests, elastic, plastic, and total strain data points are plotted versus life, and fitted lines are also shown. The total strain is given as:

$$\begin{aligned}\varepsilon_a &= \varepsilon_{ea} + \varepsilon_{pa} \\ &= \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}}\end{aligned}$$

The total curve is given by:

$$\varepsilon_a = \frac{\sigma_f'}{E}\left(2N_f\right)^b + \varepsilon_f'\left(2N_f\right)^c$$

As shown, the plastic part of the relationship above dominates fatigue life (endurance) at high strain amplitudes and low cycles to failure.

The elastic part of the relationship governs fatigue life (endurance) at low strain amplitude and high cycles to failure.

Material properties of the material is given in Table 14.1 of the prescribed textbook.

Estimate, without the need for optimization (approach elastic and plastic parts separately and jointly, OR use the fatigue curve given in Figure 1),

1. The endurance limits for the strain amplitude spectrum shown in Table 2 below. **[6/14 x 60 marks]**
2. The total damage. **[2/14 x 60 marks]**
3. The number of repetitions to failure. **[1/14 x 60 marks]**
4. The transition point from strain-life to stress-life, N_t and ε_{at}. **[3/14 x 60 marks]**
5. Would the number of repetitions be lower if only the stress-life curve was used in the assessment? Explain. **[2/14 x 60 marks]**

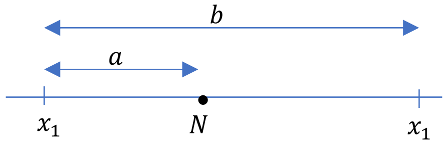
Please submit the next page with your answering sheet if you used the graphical method. Clearly indicate the levels with numbers on the graph.

For information, to interpolate on a logarithmic scale using a ruler:

$$\frac{\log_{10} N - \log_{10} x_1}{\log_{10} x_2 - \log_{10} x_1} = \frac{a}{b}$$

$$\log_{10} N = \frac{a}{b}\left(\log_{10} x_2 - \log_{10} x_1\right) + \log_{10} x_1$$

$$N = 10^{\left(\frac{a}{b}\left(\log_{10} x_2 - \log_{10} x_1\right) + \log_{10} x_1\right)}$$

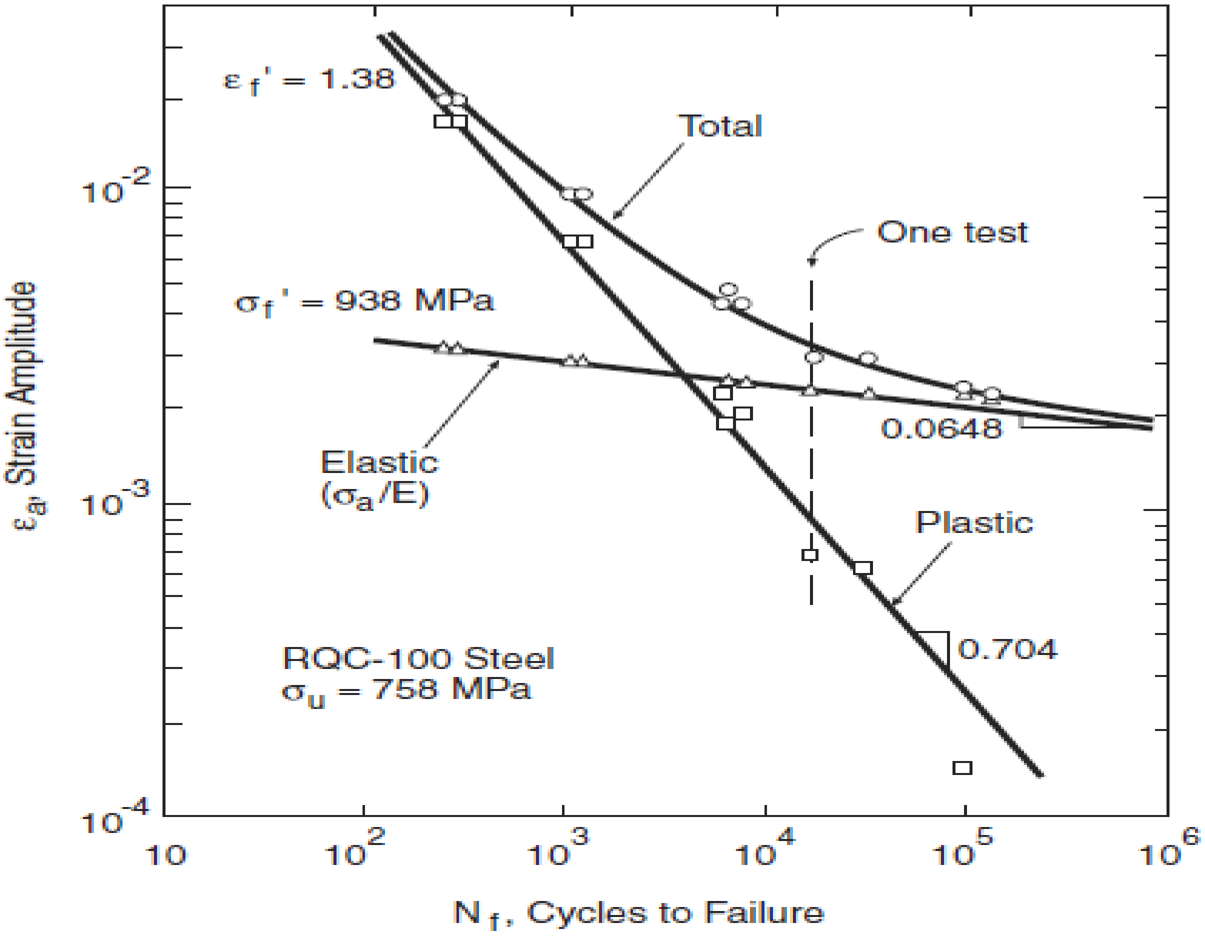


PLEASE SUBMIT THIS PAGE WITH YOUR ANSWERING SHEET

STUDENT NUMBER:____.

Table 2: Strain spectrum at a notch

ε_a	n_i	N_i	D_i
1.E-02	200		
5.E-03	1 000		
2.E-03	1 000		
	Total damage		
	Number of reptititions		



Source: (Dowling, 2013, p. 749)

Figure 1: Strain vs life curves for RQC-100 steel

2.1 Solution [scale marks to 60]

2.1.1 Endurance limits [6 marks]

In this case the endurance limits were determined from the S-N curve shown below:

For $\varepsilon_a = 0.01$:

$$\begin{aligned} N &= 10^{\left(\frac{a}{b}(\log_{10} x_2 - \log_{10} x_1) + \log_{10} x_1\right)} \\ &= 1\ 000\ cycles \end{aligned}$$

2 marks

For $\varepsilon_a = 0.005$:

$$\begin{aligned} N &= 10^{\left(\frac{a}{b}(\log_{10} x_2 - \log_{10} x_1) + \log_{10} x_1\right)} \\ &= 10^{\left(\frac{20}{30}(\log 10^4 - \log 10^3) + \log 10^3\right)} \\ &= 4\ 600\ cycles \end{aligned}$$

2 marks

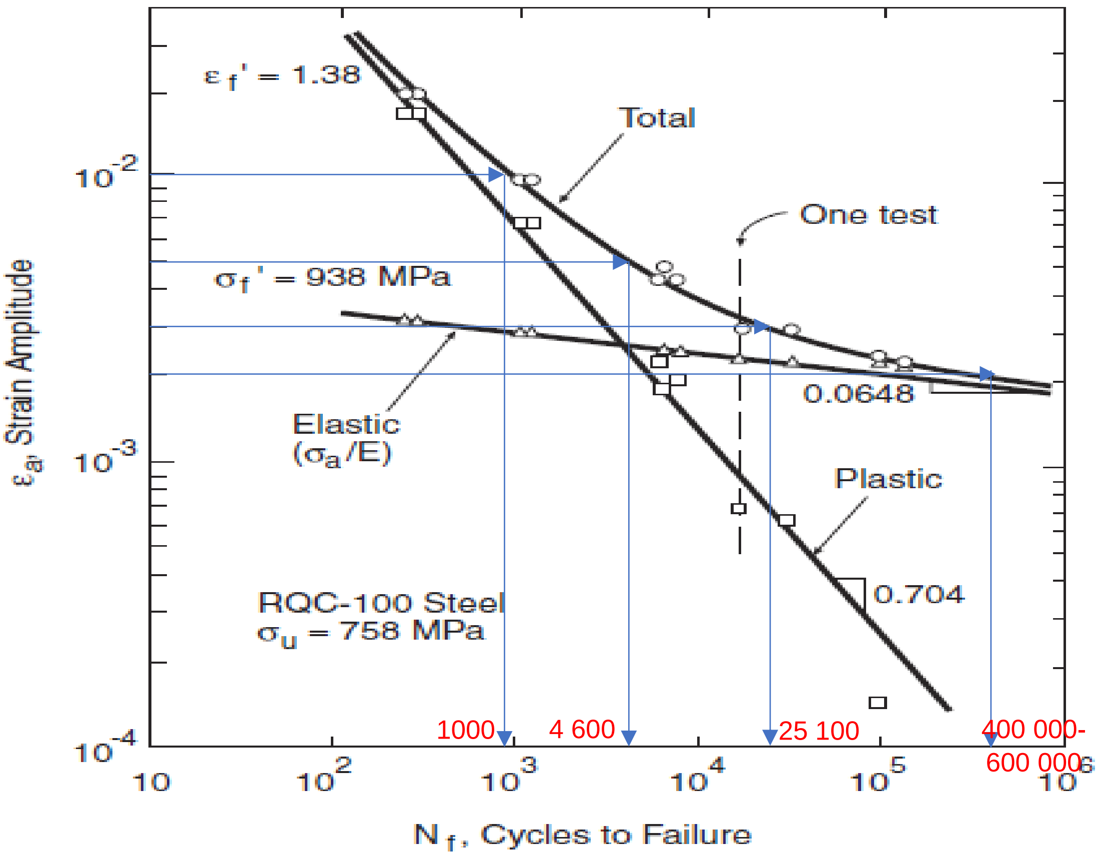
For $\varepsilon_a = 0.003$:

$$\begin{aligned} N &= 10^{\left(\frac{a}{b}(\log_{10} x_2 - \log_{10} x_1) + \log_{10} x_1\right)} \\ &= 10^{\left(\frac{12}{30}(\log 10^5 - \log 10^4) + \log 10^4\right)} \\ &= 25\ 100\ cycles \end{aligned}$$

For $\varepsilon_a = 0.002$:

$$\begin{aligned} N &= 10^{\left(\frac{a}{b}(\log_{10} x_2 - \log_{10} x_1) + \log_{10} x_1\right)} \\ &= 10^{\left(\frac{15}{25}(\log 10^6 - \log 10^5) + \log 10^5\right)} \\ &= 400\ 000\ cycles \end{aligned}$$

2 marks



2.1.2 Total damage [2 marks]

The total damage is:

2 marks

$$
\begin{aligned}
D &= \frac{200}{1000} + \frac{1000}{4600} + \frac{1000}{400\,000} \\
&= 0.42
\end{aligned}
$$

2.1.3 Number of repetitions to failure [1 mark]

The number of repetitions to failure is:

$$
\begin{aligned}
B_f &= \frac{1}{D} \\
&= 2.4
\end{aligned}
$$

1 mark

CHECK CALCULATIONS IN EXCEL

ε_a	n_i	N_i	D_i
1.E-02	200	1000	0.2
5.E-03	1 000	4600	0.217391
2.E-03	1 000	316000	0.003165
		Total damage	0.420556
		Number of reptititions	2.377805

2.1.4 Transition point [3 marks]

The transition point on the fatigue curve is where the parts of the elastic and plastic components are equal to each other. Therefore:

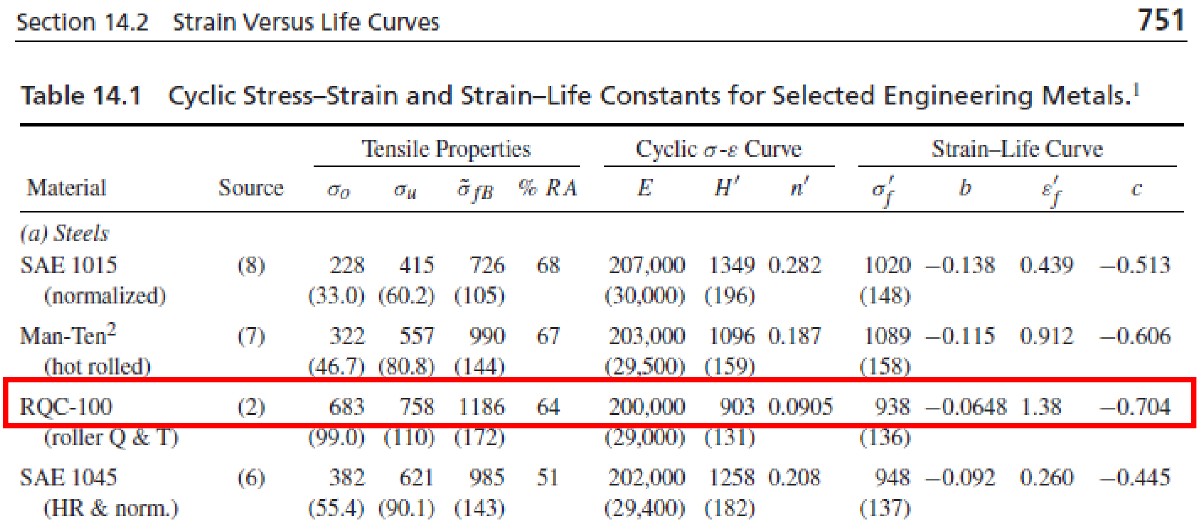
$$
\begin{aligned}
\frac{\sigma_f'}{E}(2N_t)^b &= \varepsilon_f(2N_t)^c \\
(2N_t)^{b-c} &= \frac{\varepsilon_f' E}{\sigma_f'} \\
N_t &= \frac{\left(\frac{\varepsilon_f' E}{\sigma_f'}\right)^{\frac{1}{b-c}}}{2} \\
&= \frac{\left(\frac{1.38 \times 200\,000}{938}\right)^{\frac{1}{-0.0648+0.704}}}{2} \\
&= 3\,640\ cycles
\end{aligned}
$$

2 marks

At this endurance, the transition strain is:

$$
\begin{aligned}
\varepsilon_{at} &= 2 \times \frac{\sigma_f'}{E}(2N_t)^b \\
&= 0.0053
\end{aligned}
$$

1 mark



Section 14.2 Strain Versus Life Curves 751

Table 14.1 Cyclic Stress–Strain and Strain–Life Constants for Selected Engineering Metals.[1]

		Tensile Properties				Cyclic σ-ε Curve			Strain–Life Curve			
Material	Source	σ_o	σ_u	$\tilde{\sigma}_{fB}$	% RA	E	H'	n'	σ'_f	b	ε'_f	c
(a) Steels												
SAE 1015	(8)	228	415	726	68	207,000	1349	0.282	1020	−0.138	0.439	−0.513
(normalized)		(33.0)	(60.2)	(105)		(30,000)	(196)		(148)			
Man-Ten[2]	(7)	322	557	990	67	203,000	1096	0.187	1089	−0.115	0.912	−0.606
(hot rolled)		(46.7)	(80.8)	(144)		(29,500)	(159)		(158)			
RQC-100	(2)	683	758	1186	64	200,000	903	0.0905	938	−0.0648	1.38	−0.704
(roller Q & T)		(99.0)	(110)	(172)		(29,000)	(131)		(136)			
SAE 1045	(6)	382	621	985	51	202,000	1258	0.208	948	−0.092	0.260	−0.445
(HR & norm.)		(55.4)	(90.1)	(143)		(29,400)	(182)		(137)			

2.1.5 Stress-life only [2 marks]

If only the stress-life fatigue curve was used, the endurances would have been less at the same strain amplitudes, the damage would have been higher and the number of repetitions smaller.

The stress-life curve is more suitable for high cycle fatigue analysis using linear elastic stress analysis. To compensate for the lower sensitivity for surface effects at the high stress and low cycles, the fatigue notch factor with different sensitivity to the notch at high and low cycles.

The lower fatigue notch factor sensitivity at high stress also makes provision for the reduction in stress due to plasticity. Still, the stress-life approach is overly conservative for highly stressed notches.

Mark as follows:

2 marks if the student demonstrates understanding of the answer.

They will provide different answers – mark accordingly.

1 mark for a student who answered “yes” with no logic explanation.

3 WELD FATIGUE [60 Marks – one mark/minute]

Note, to obtain maximum points for each problem clearly motivate solutions and equations used.

The steel H-beams shown below are joined by a complete joint penetration butt weld (made from both sides) with no cope holes. The steel has yield strength 350 MPa and ultimate tensile strength 490 MPa and welding was done with a perfectly matched electrode.

The web and flange thickness is 35 mm.

The height of the weld convexity is less than 10% of the weld width with smooth transition to the plate surface.

Weld run-on and run-off pieces were used during welding and removed. The edges were then ground flush in the direction of the stress. An NDT inspection revealed no defects in the weld.

The operating temperature is 20 °C and the surface is corrosion protected.

Your class notes have extractions of some detail categories from BS EN 1993-1-9.

Please answer the following:

1. What is the fatigue life of the component, in years, for the load spectrum given below? Assume a high consequence of failure and damage tolerant design **[55 Marks]**
2. Will this detail category benefit from weld toe dressing? **[5 Marks]**

Table 3: Stress spectrum for a period of 10 days

$\Delta\sigma_R$ [MPa]	n_i
110	200
60	2 000
25	2 000 000

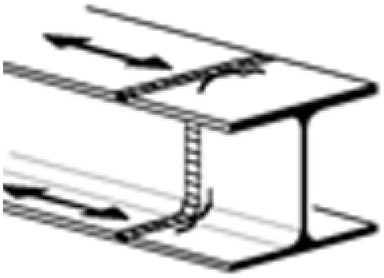


Figure 2: H-beam under axial loading

3.1 Answers

3.1.1 Question 1 (16 minutes – allow 35 minutes) [55 Marks – one mark/minute]

3.1.1.1 *Partial factor for fatigue*

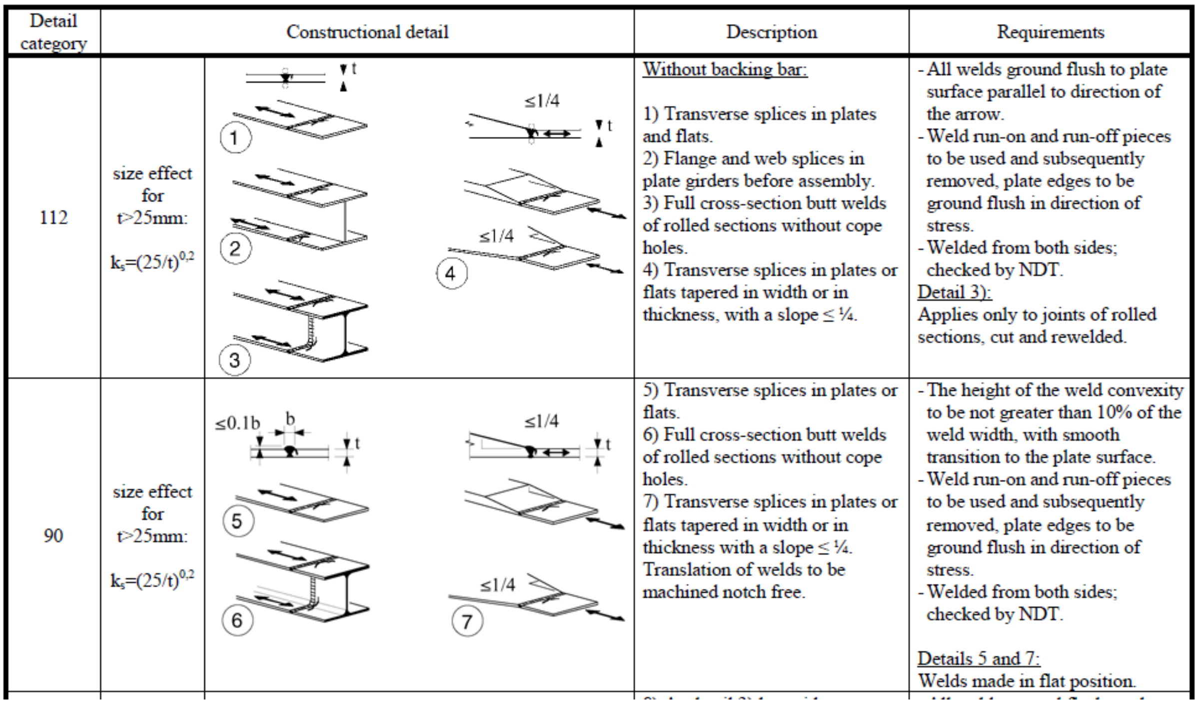
From the formula sheet the partial factor for fatigue for high consequence of failure and damage tolerant assessment method is 1.15.

Assessment method	**Consequence of failure**	
	Low consequence	**High consequence**
Damage tolerant	1.00	**1.15**
Safe life	1.15	1.35
Source: BS EN 1993-1-9, 2005:11		

3.1.1.2 *Detail category*

The detail category is 90 because the welds were not all ground flush to the plate.

Table 8.3: Transverse butt welds



Detail category	Constructional detail		Description	Requirements
112	size effect for t>25mm: $k_s=(25/t)^{0.2}$		Without backing bar: 1) Transverse splices in plates and flats. 2) Flange and web splices in plate girders before assembly. 3) Full cross-section butt welds of rolled sections without cope holes. 4) Transverse splices in plates or flats tapered in width or in thickness, with a slope ≤ ¼.	- All welds ground flush to plate surface parallel to direction of the arrow. - Weld run-on and run-off pieces to be used and subsequently removed, plate edges to be ground flush in direction of stress. - Welded from both sides; checked by NDT. Detail 3): Applies only to joints of rolled sections, cut and rewelded.
90	size effect for t>25mm: $k_s=(25/t)^{0.2}$		5) Transverse splices in plates or flats. 6) Full cross-section butt welds of rolled sections without cope holes. 7) Transverse splices in plates or flats tapered in width or in thickness with a slope ≤ ¼. Translation of welds to be machined notch free.	- The height of the weld convexity to be not greater than 10% of the weld width, with smooth transition to the plate surface. - Weld run-on and run-off pieces to be used and subsequently removed, plate edges to be ground flush in direction of stress. - Welded from both sides; checked by NDT. Details 5 and 7: Welds made in flat position.

3.1.1.3 *Thickness compensation*

The factor to compensate for thickness more than 25 mm is:

$$\begin{aligned} k_s &= \left(\frac{25}{t}\right)^{0.2} \\ &= \left(\frac{25}{35}\right)^{0.2} \\ &= 0.93 \end{aligned}$$

3.1.1.4 *Temperature compensation*

At room temperature $C_T = 1.0$.

3.1.1.5 *Constant amplitude fatigue limit*

The characteristic strength is given at 2 million cycles, and the constant amplitude fatigue limit at 5 million cycles. Therefore, the thickness modified constant amplitude fatigue limit is:

$$\begin{aligned}\Delta\sigma_D &= \left(\frac{2}{5}\right)^{\frac{1}{3}}\left(\frac{\Delta\sigma_C}{\gamma_F}\right)k_s \\ &= \left(\frac{2}{5}\right)^{\frac{1}{3}} \times \frac{90}{1.15} \times 0.93 \\ &= 53.6\ MPa\end{aligned}$$

3.1.1.6 Cut-off limit

The cut-off limit is:

$$\begin{aligned}\Delta\sigma_L &= \left(\frac{5}{100}\right)^{\frac{1}{5}}\Delta\sigma_D \\ &= \left(\frac{5}{100}\right)^{\frac{1}{5}} 53.6 \\ &= 29.4\ MPa\end{aligned}$$

3.1.1.7 S_r-N curve

The fatigue curve for the weld detail is then shown below from which the endurance at any stress range can be calculated:

$$N_R = \begin{cases}\left(\frac{\Delta\sigma_C}{\Delta\sigma_R}\right)^{m_1} N_C & m_1 = 3 \quad for\ 1.5f_y > \Delta\sigma_R \geq \Delta\sigma_D \\ \left(\frac{\Delta\sigma_D}{\Delta\sigma_R}\right)^{m_2} N_D & m_2 = 5 \quad for\ \Delta\sigma_D > \Delta\sigma_R \geq \Delta\sigma_L \\ \infty & \quad for\ \Delta\sigma_R < \Delta\sigma_L\end{cases}$$

3.1.1.8 Calculations

For $\boldsymbol{\Delta\sigma_R = 110, n_1 = 200}$

The endurance at this stress range is then:

$$\begin{aligned}N_R &= \left(\frac{\Delta\sigma_D}{\Delta\sigma_R}\right)^{m_1} N_D \\ &= \left(\frac{53.6}{110}\right)^3 \times 5 \times 10^6 \\ &= 578\,480\ cycles\end{aligned}$$

The damage is then: $D_1 = \frac{200}{578480} = 3.46 \times 10^{-4}$

For $\boldsymbol{\Delta\sigma_R = 60, n_2 = 2\,000}$

The endurance is:

$$\begin{aligned}N_R &= \left(\frac{\Delta\sigma_D}{\Delta\sigma_R}\right)^3 N_D \\ &= \left(\frac{53.6}{60}\right)^3 \times 5 \times 10^6 \\ &= 3\,564\,600\end{aligned}$$

The damage is then:

$$\begin{aligned}D_2 &= \frac{2\,000}{3\,564\,600} \\ &= 5.61 \times 10^{-4}\end{aligned}$$

For $\boldsymbol{\Delta\sigma_R = 25, n_2 = 2\,000}$

This stress range is below the cut-off limit and will not do any damage. Therefore:

$$D_3 = 0$$

3.1.1.9 *Total damage and fatigue life*

The total damage over 10 days is:

$$\begin{aligned} D_{total} &= 5.61 \times 10^{-4} + 3.46 \times 10^{-4} \\ &= 9.07 \times 10^{-4} \end{aligned}$$

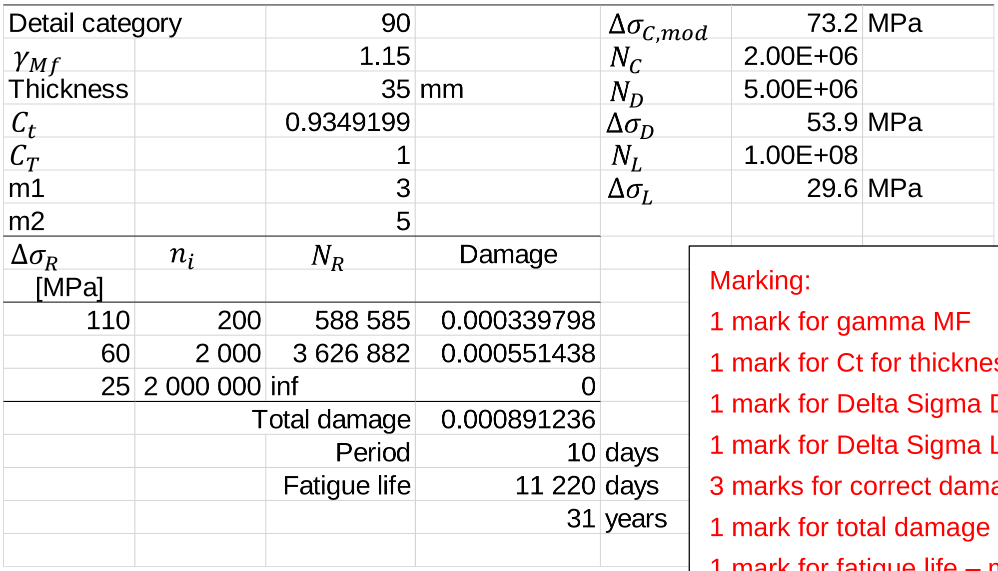
The number of repetitions to 5% probability of crack initiation in logN is then:

$$\begin{aligned} B_f &= \frac{1}{D_{total}} \\ &= 1\,102.5 \end{aligned}$$

If one repetition was 10 days, then the fatigue life in days is:

$$\begin{aligned} L &= 10 \times B_f \\ &= 11\,025\ days \\ &= 30.2\ years \end{aligned}$$

3.1.1.10 *Verification in Excel*



Detail category		90		$\Delta\sigma_{C,mod}$	73.2	MPa
γ_{Mf}		1.15		N_C	2.00E+06	
Thickness		35	mm	N_D	5.00E+06	
C_t		0.9349199		$\Delta\sigma_D$	53.9	MPa
C_T		1		N_L	1.00E+08	
m1		3		$\Delta\sigma_L$	29.6	MPa
m2		5				
$\Delta\sigma_R$ [MPa]	n_i	N_R	Damage			
110	200	588 585	0.000339798			
60	2 000	3 626 882	0.000551438			
25	2 000 000	inf	0			
		Total damage	0.000891236			
		Period	10	days		
		Fatigue life	11 220	days		
			31	years		

Marking:

1 mark for gamma MF

1 mark for Ct for thickness

1 mark for Delta Sigma D

1 mark for Delta Sigma L

3 marks for correct damage

1 mark for total damage – mark with error

1 mark for fatigue life – mark with error

3.1.2 Question 2 [5 Marks – one mark/minute]

NO, in this case the surface was ground flush at the edges, and all stress concentrations were removed. Then weld toe dressing will not necessarily improve the situation at the edges.

The weld detail will benefit from post-weld toe dressing if it was not ground because the weld toe is the point of expected crack initiation. The nominal stress is also transverse to the weld.

Marking:

100% for motivation of answer. Consider both as acceptable answers if properly motivated – taking into consideration that the weld is ground.

2 marks for yes as answer with motivation.

4 References

Dowling, N. (2013). *Mechanical Behavior of Materials: Engineering Methods for Deformation, Fracture, and Fatigue* (4th ed.). Boston: Pearson.

FORMULA SHEET: Fatigue, Fracture Mechanics

4.1 Structural Mechanics

$$\sigma_i = \frac{M_x y_i}{I_{xx}} - \frac{M_y x_i}{I_{yy}} + \frac{F}{A}$$

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}\left(\sigma_y + \sigma_z\right) + \alpha \Delta T$$

4.2 Fatigue: Machined Components

$$\sigma_{ar}^{m} N = C$$

$$\sigma_{ar,1}^{m} N_1 = \sigma_{ar,2}^{m} N_2$$

$$m = \frac{\log\left(\frac{N_2}{N_1}\right)}{\log\left(\frac{\sigma_{ar,1}}{\sigma_{ar,2}}\right)}$$

$$N_R = \begin{cases} \left(\frac{\sigma_{ar,1}}{\sigma_{ar}}\right)^{m} N_1 & 0.9 f_{ut} \geq \sigma_{ar} \geq S_e \\ \infty & \sigma_{ar} < S_e \end{cases}$$

OR

$$\sigma_{ar} N^{b} = C$$

$$\sigma_{ar,1} N_1^{b} = \sigma_{ar,2} N_2^{b}$$

$$b = \frac{\log\left(\frac{\sigma_{ar,1}}{\sigma_{ar,2}}\right)}{\log\left(\frac{N_2}{N_1}\right)}$$

$$N_R = \begin{cases} \left(\frac{\sigma_{ar,1}}{\sigma_{ar}}\right)^{\frac{1}{b}} N_1 & 0.9 f_{ut} \geq \sigma_{ar} \geq S_e \\ \infty & \sigma_{ar} < S_e \end{cases}$$

Endurance limit estimates:

$$\sigma_{erb} = \begin{cases} 0.25 BHN\ ksi & for\ BHN \leq 400 \\ 100\ ksi & for\ BHN > 400 \end{cases}$$

Steel

$$\sigma_{erb} = \begin{cases} 0.5\sigma_u & for\ \sigma_u \leq 200\ ksi\ (1\,400\ MPa) \\ 100\ ksi\ (700\ MPa) & for\ \sigma_u > 200\ ksi\ (1\,400\ MPa) \end{cases}$$

Cast Iron + Cast Steels:

$$\sigma_{erb} = \begin{cases} 0.45\sigma_u & for\ \sigma_u \leq 600\ MPa \\ 275\ MPa & for\ \sigma_u > 600\ MPa \end{cases}$$

Stress concentrations

$$\sigma_{sc} = K_t S$$

$$K_t = \frac{\sigma_{sc}}{S}$$

$$K_f = \frac{\sigma_{erb}^{(un-notched)}}{\sigma_{erb}^{(nothced)}}$$

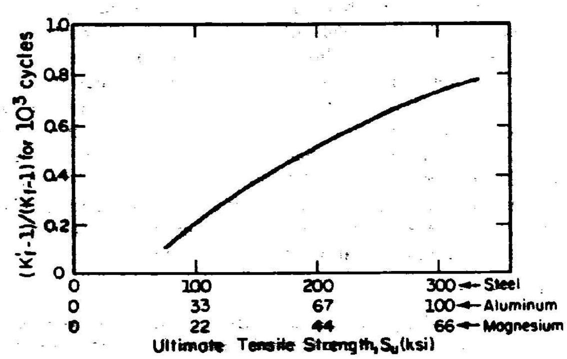
$$K_f = 1 + \frac{K_t - 1}{\left(1 + \frac{a}{r}\right)}$$

$$a = \left[\frac{300}{\sigma_u[ksi]}\right]^{1.8} \times 10^{-3}\ in.$$

$$\frac{K_f' - 1}{K_f - 1} = f(\sigma_u) = q$$

$$\sigma_{arb,1000}' = \frac{\sigma_{arb,1000}}{K_f'}$$

$$\sigma_{erb}' = \frac{\sigma_{erb}}{K_f}$$



Mean stress:

Approach	Equations
Modified Goodman	$\sigma_{ar} = \frac{\sigma_a}{1 - \frac{\sigma_m}{\sigma_u}}$
Gerber	$\sigma_{ar} = \frac{\sigma_a}{1 - \left(\frac{\sigma_m}{\sigma_u}\right)^2}, for\ \sigma_m \geq 0$
Morrow	$\sigma_{ar} = \frac{\sigma_a}{1 - \frac{\sigma_m}{\sigma_f'}}$
SWT	$\sigma_{ar} = \sqrt{\sigma_{max}\sigma_a}$ $= \sqrt{(\sigma_m + \sigma_a)\sigma_a}$ $= \sigma_{max}\sqrt{\frac{1-R}{2}}$
Walker	$\sigma_{ar} = \sigma_{max}^{1-\gamma}\sigma_a^{\gamma} \quad (\sigma_{max} > 0)$ $= \sigma_{max}\left(\frac{1-R}{2}\right)^{\gamma} \quad (\sigma_{max} > 0)$ $\gamma = -0.000200\sigma_u + 0.8818 \quad (\sigma_u\ in\ MPa)$

Modifying factors

$$\sigma_{er} = \sigma_{\mathrm{erb}}' C_{size} C_{load} C_{surf} C_T C_{rel}$$

$$\sigma_{erb}' = \frac{0.5\sigma_u}{K_f}$$

$$\sigma_{ar,10^3} = \sigma'_{arb,10^3} C_{load} C_T C_{rel}$$

$$\sigma'_{arb,10^3} = \frac{0.9\sigma_u}{K'_f}$$

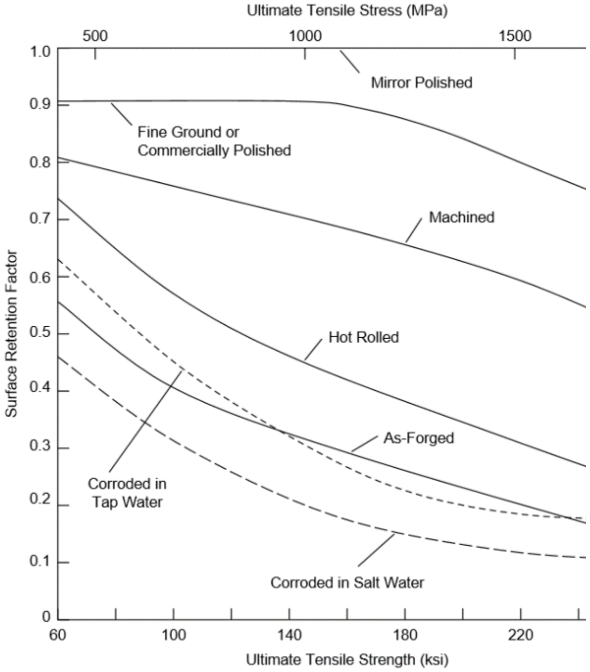
Size:

$$C_{size} = \begin{cases} 1.0, if\ d \le 8\ mm \\ 1.189 d^{-0.097}, if\ 8\ mm < d \le 250\ mm \end{cases}$$

Load:

$$\sigma_{er,axial} = 0.70\sigma_{erb}$$

C_{load} = 0.7 for axial loading if the fatigue S-N curve was obtained from completely reversed loading.



Temperature:

$$C_T = \begin{cases} 1.0\ for\ T \le 450\ ℃ \\ 1 - 5.8^{-3}(T - 450),\ for\ 450 < T \le 550\ ℃ \end{cases}$$

Reliability:

Reliability $1 - p_f$	C_r
0.5	1
0.9	0.897
0.95	0.868
0.99	0.814
0.999	0.753
0.9999	0.702
0.99999	0.659
0.999999	0.620

4.3 Fatigue: Large scale manufactured components

$$N_R = \begin{cases} \left(\frac{\Delta\sigma_D}{\Delta\sigma_R}\right)^{m_1} N_D & m_1 = 3 & 1.5 f_y > \Delta\sigma_R \ge \Delta\sigma_D \\ \left(\frac{\Delta\sigma_D}{\Delta\sigma_R}\right)^{m_2} N_D & m_2 = 5 & \Delta\sigma_D > \Delta\sigma_R \ge \Delta\sigma_L \\ \infty & & \Delta\sigma_R < \Delta\sigma_L \end{cases}$$

$$N_C = 2 \times 10^6\ cycles$$
$$N_D = 5 \times 10^6\ cycles$$
$$N_L = 100 \times 10^6\ cycles$$

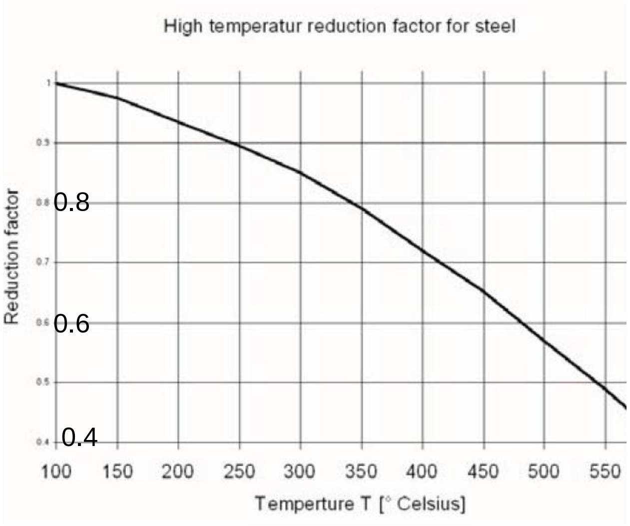
$$\Delta\sigma_{C,mod} = \frac{\Delta\sigma_C}{\gamma_{Mf}} C_t C_T C_{PWT}$$

Partial factor for fatigue:

Assessment	Consequence of failure	
	Low	High
Damage tolerant	1.0	1.15
Safe life	1.15	1.35

Temperature:

$$\Delta_{C,HT} = \Delta\sigma_C \frac{E_{HT}}{E_{20°C}}$$



<u>Grinding & TIG dressing:</u>

Effective stress range: $\Delta\sigma = \Delta\sigma$

Steel:

$$\Delta\sigma_C = min\begin{cases} 1.3 \times \Delta\sigma_C \\ 112 \end{cases}$$

Aluminium:

$$\Delta\sigma_C = min\begin{cases} 1.3 \times \Delta\sigma_C \\ 45 \end{cases}$$

<u>Peening:</u>

Effective stress range:

$$\Delta\sigma = \begin{cases} \Delta\sigma & R > 0 \\ \sigma_{max} & 0 < R \leq 0.4 \\ No\ benefit & R > 0.4 \end{cases}$$

$$\Delta\sigma_{c,mod} = \begin{cases} Steel & f_y < 355 & \min(1.3\Delta\sigma_c, 112) \\ Steel & f_y \geq 355 & \min(1.6\Delta\sigma_c, 125) \\ Al & & \min(1.6\Delta\sigma_c, 56) \end{cases}$$

4.4 Fracture Mechanics

Universal equation: $K = \beta\sigma\sqrt{\pi a}$

Plastic collapse: $F_{pc} = \frac{f_y}{A_{nett}}$

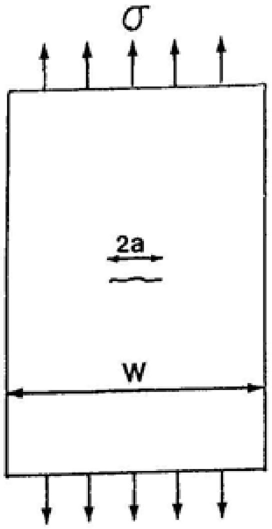
Fracture:

$$K = K_{Ic}$$
$$K_{Ic} = \beta\sigma\sqrt{\pi a_f}$$
$$a_f = \frac{1}{\pi}\left(\frac{K_{Ic}}{\beta\sigma}\right)^2$$
$$\sigma_f = \frac{K_{Ic}}{\beta\sqrt{\pi a_{fr}}}$$

$$a_{cri} = \min\begin{cases} a_{pc} & for\ plastic\ collapse \\ a_f & for\ fracture \end{cases}$$

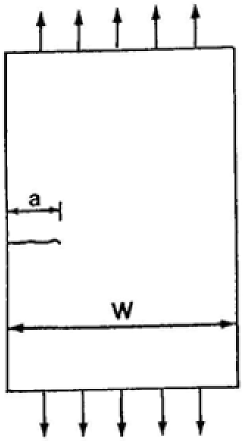
Stress concentration factors:

Centre cracked plate: $\beta = 1 + 0.256\left(\frac{a}{W}\right) - 1.152\left(\frac{a}{W}\right)^2 + 12.2\left(\frac{a}{W}\right)^3$



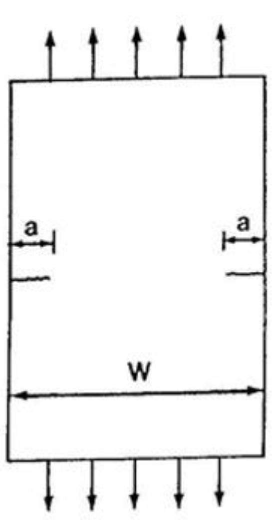
Single edge crack:

$$\beta = 1.12 - 0.23\left(\frac{a}{W}\right) + 10.56\left(\frac{a}{W}\right)^2 - 21.74\left(\frac{a}{W}\right)^3 + 30.42\left(\frac{a}{W}\right)^4$$



Double edge crack:

$$\beta = 1.12 + 0.43\left(\frac{a}{W}\right) - 4.79\left(\frac{a}{W}\right)^2 + 15.46\left(\frac{a}{W}\right)^3$$



LEFM

$$B, W - a, a \geq 2.5\left(\frac{K_{Ic}}{f_{yt}}\right)^2$$

$$W \geq 5.0\left(\frac{K_{Ic}}{f_{yt}}\right)^2$$

Fracture toughness estimation: $K_{IC} = 11.4\sqrt{C_v}$

Lower limit: $K_{IC} = 21.6(C_v)^{0.17}$

Note, C_v in Joule, K_{IC} in $MPa.\sqrt{m}$

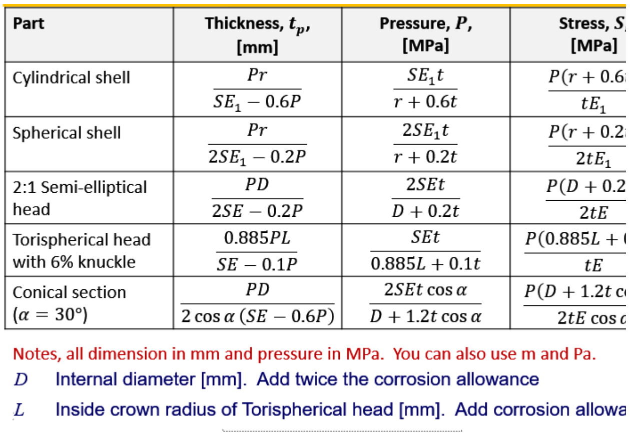
Crack growth

$$\frac{da}{dN} = C_p(\Delta K^+)^{m_p}$$

$$\int_0^N dN = \int_{a_i}^{a_e} \frac{1}{C_p\left(\beta\Delta\sigma\sqrt{\pi a}\right)^{m_p}} da$$

4.5 Pressure equipment

$$P_{des} = P + \rho g h$$



Part	Thickness, t_p, [mm]	Pressure, P, [MPa]	Stress, S, [MPa]
Cylindrical shell	$\frac{Pr}{SE_1 - 0.6P}$	$\frac{SE_1 t}{r + 0.6t}$	$\frac{P(r + 0.6}{tE_1}$
Spherical shell	$\frac{Pr}{2SE_1 - 0.2P}$	$\frac{2SE_1 t}{r + 0.2t}$	$\frac{P(r + 0.2}{2tE_1}$
2:1 Semi-elliptical head	$\frac{PD}{2SE - 0.2P}$	$\frac{2SEt}{D + 0.2t}$	$\frac{P(D + 0.2}{2tE}$
Torispherical head with 6% knuckle	$\frac{0.885PL}{SE - 0.1P}$	$\frac{SEt}{0.885L + 0.1t}$	$\frac{P(0.885L +}{tE}$
Conical section ($\alpha = 30°$)	$\frac{PD}{2\cos\alpha\,(SE - 0.6P)}$	$\frac{2SEt\cos\alpha}{D + 1.2t\cos\alpha}$	$\frac{P(D + 1.2t\ c}{2tE\cos}$

Notes, all dimension in mm and pressure in MPa. You can also use m and Pa.

D Internal diameter [mm]. Add twice the corrosion allowance

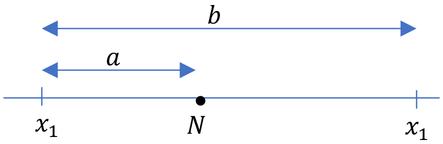
L Inside crown radius of Torispherical head [mm]. Add corrosion allowa

4.6 Interpolation on a logarithmic scale using your ruler

$$\frac{\log_{10}N - \log_{10}x_1}{\log_{10}x_2 - \log_{10}x_1} = \frac{a}{b}$$

$$\log_{10}N = \frac{a}{b}(\log_{10}x_2 - \log_{10}x_1) + \log_{10}x_1$$

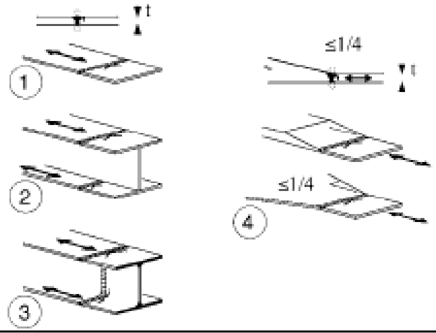
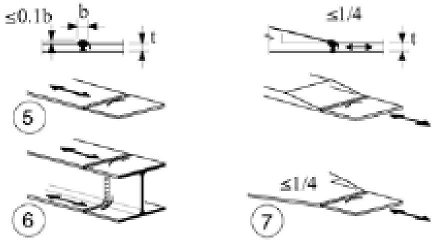
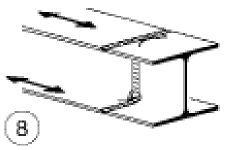
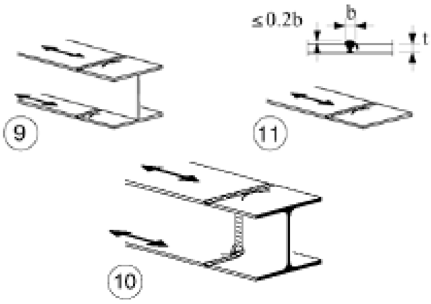
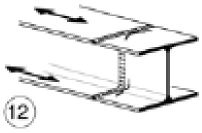
$$N = 10^{\left(\frac{a}{b}(\log_{10}x_2 - \log_{10}x_1) + \log_{10}x_1\right)}$$



CONVERSIONS

1 ksi = 6.89 MPa

4.7 Detail catgories

Detail category	Constructional detail		Description	Requirements
112	size effect for $t>25$mm: $k_s=(25/t)^{0,2}$		Without backing bar: 1) Transverse splices in plates and flats. 2) Flange and web splices in plate girders before assembly. 3) Full cross-section butt welds of rolled sections without cope holes. 4) Transverse splices in plates or flats tapered in width or in thickness, with a slope ≤ ¼.	- All welds ground flush to plate surface parallel to direction of the arrow. - Weld run-on and run-off pieces to be used and subsequently removed, plate edges to be ground flush in direction of stress. - Welded from both sides; checked by NDT. Detail 3): Applies only to joints of rolled sections, cut and rewelded.
90	size effect for $t>25$mm: $k_s=(25/t)^{0,2}$		5) Transverse splices in plates or flats. 6) Full cross-section butt welds of rolled sections without cope holes. 7) Transverse splices in plates or flats tapered in width or in thickness with a slope ≤ ¼. Translation of welds to be machined notch free.	- The height of the weld convexity to be not greater than 10% of the weld width, with smooth transition to the plate surface. - Weld run-on and run-off pieces to be used and subsequently removed, plate edges to be ground flush in direction of stress. - Welded from both sides; checked by NDT. Details 5 and 7: Welds made in flat position.
90	size effect for $t>25$mm: $k_s=(25/t)^{0,2}$		8) As detail 3) but with cope holes.	- All welds ground flush to plate surface parallel to direction of the arrow. - Weld run-on and run-off pieces to be used and subsequently removed, plate edges to be ground flush in direction of stress. - Welded from both sides; checked by NDT. - Rolled sections with the same dimensions without tolerance differences
80	size effect for $t>25$mm: $k_s=(25/t)^{0,2}$		9) Transverse splices in welded plate girders without cope hole. 10) Full cross-section butt welds of rolled sections with cope holes. 11) Transverse splices in plates, flats, rolled sections or plate girders.	- The height of the weld convexity to be not greater than 20% of the weld width, with smooth transition to the plate surface. - Weld not ground flush - Weld run-on and run-off pieces to be used and subsequently removed, plate edges to be ground flush in direction of stress. - Welded from both sides; checked by NDT. Detail 10: The height of the weld convexity to be not greater than 10% of the weld width, with smooth transition to the plate surface.
63			12) Full cross-section butt welds of rolled sections without cope hole.	- Weld run-on and run-off pieces to be used and subsequently removed, plate edges to be ground flush in direction of stress. - Welded from both sides.

4.8 Joint efficiencies

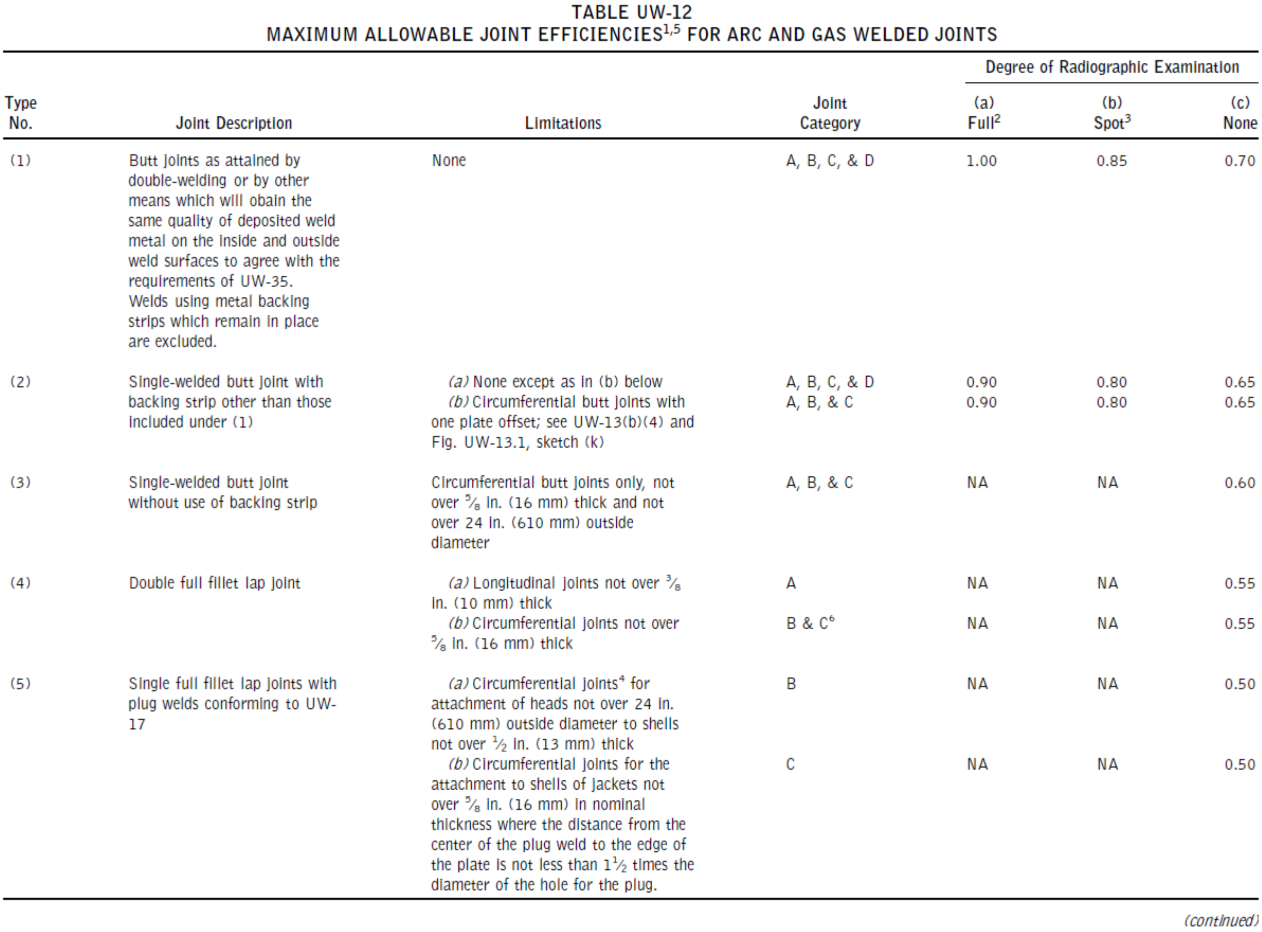


TABLE UW-12

MAXIMUM ALLOWABLE JOINT EFFICIENCIES[1,5] FOR ARC AND GAS WELDED JOINTS

				Degree of Radiographic Examination		
Type No.	Joint Description	Limitations	Joint Category	(a) Full[2]	(b) Spot[3]	(c) None
(1)	Butt joints as attained by double-welding or by other means which will obain the same quality of deposited weld metal on the inside and outside weld surfaces to agree with the requirements of UW-35. Welds using metal backing strips which remain in place are excluded.	None	A, B, C, & D	1.00	0.85	0.70
(2)	Single-welded butt joint with backing strip other than those included under (1)	*(a)* None except as in (b) below	A, B, C, & D	0.90	0.80	0.65
		(b) Circumferential butt joints with one plate offset; see UW-13(b)(4) and Fig. UW-13.1, sketch (k)	A, B, & C	0.90	0.80	0.65
(3)	Single-welded butt joint without use of backing strip	Circumferential butt joints only, not over 5/8 in. (16 mm) thick and not over 24 in. (610 mm) outside diameter	A, B, & C	NA	NA	0.60
(4)	Double full fillet lap joint	*(a)* Longitudinal joints not over 3/8 in. (10 mm) thick	A	NA	NA	0.55
		(b) Circumferential joints not over 5/8 in. (16 mm) thick	B & C[6]	NA	NA	0.55
(5)	Single full fillet lap joints with plug welds conforming to UW-17	*(a)* Circumferential joints[4] for attachment of heads not over 24 in. (610 mm) outside diameter to shells not over 1/2 in. (13 mm) thick	B	NA	NA	0.50
		(b) Circumferential joints for the attachment to shells of jackets not over 5/8 in. (16 mm) in nominal thickness where the distance from the center of the plug weld to the edge of the plate is not less than 1 1/2 times the diameter of the hole for the plug.	C	NA	NA	0.50

(continued)